A Theory of Stable Price Dispersion

David P. Myatt
London Business School
www.david-p-myatt.org
dmyatt@london.edu

David Ronayne
University of Oxford
www.davidronayne.com
david.ronayne@economics.ox.ac.uk

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Abstract. We propose a two-stage replacement for established "clearinghouse" pricing models: second-stage retail prices are constrained by first-stage list prices. In contrast to the mixed-strategy equilibria of single-stage games, a unique profile of distinct prices is supported by the play of pure strategies along the equilibrium path, and so we predict stable price dispersion. We find novel results in applications of our approach to models of sales, product prominence, advertising, and costly buyer search.

Keywords: price dispersion, clearinghouse models, prominence, advertising, buyer search.

1. Introduction

Seemingly identical products are often sold at different prices by several firms. The "clearinghouse" pricing framework, associated with Varian's (1980) "model of sales" and other contributions (Shilony, 1977; Rosenthal, 1980; Narasimhan, 1988), generates price dispersion via the mixed-strategy equilibria of a single-stage pricing game. Such equilibria are naturally susceptible to ex post deviations and predict dynamically uncorrelated variation in firms' prices. This is inconsistent with any ability of retailers to offer rapid discounts in response to others, and with the empirical lack of sufficiently frequent temporal variation in otherwise disperse prices.

We suggest a two-stage alternative that produces stable price dispersion. In a first stage, firms post list prices. In a second stage, they may discount (but not raise) those prices. Under several specifications, we find a unique set of prices that are supported by the equilibrium play of pure strategies. Firms' prices differ (even for symmetric firms) and the opportunity to offer a second-stage discount is not used (but the possibility determines equilibrium prices). For asymmetric firms we pin down which supplier charges each price. We develop and apply our approach using a suite of three further models: we relate pricing positions to firms' prominence; we characterize firms' choice of advertising exposure; and we study the search decisions of buyers. The results suggest that our approach succeeds as a tractable replacement for the clearinghouse pricing stage of models in which buyers have incomplete or costly access to prices.

Before explaining our approach and the results that flow from it, we pause to describe the price dispersion that we seek to explain and the empirical justification for our assumptions.

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Empirical studies have identified extensive price dispersion. For example, Kaplan and Menzio (2015) used the large Kilts-Nielsen panel of 50,000 households to show that the standard deviation (relative to the mean) of prices at brick-and-mortar stores ranges from 19% (when products are defined narrowly) to 36% (when defined broadly). However, only a minority of variation can be attributed to intertemporal price changes.[2] This differs from a literal interpretation of the symmetric mixed equilibrium of a clearinghouse model.[3] More recently, Aas, Wulfsberg, and Moen (2018) found high persistence of price dispersion: the same price typically lasts 1–3 months but many last for over a year, and stores charging prices in a particular quartile of the distribution tend to stay there with high probability (0.8–0.9) month-to-month. Relatedly, Gorodnichenko, Sheremirov, and Talavera (2018) examined daily online pricing data. They reported (pp. 1764–1766) that "although online prices change more frequently than offline prices, they nevertheless exhibit relatively long spells of fixed prices" and so "online price setting is characterized by considerable frictions." Specifically, prices are fixed for long spells of 7–20 weeks and "clearly do not adjust every instant." They concluded that prices tend to vary in the cross section rather than over time. A stylized summary of empirical findings is that firms persistently occupy different high and low pricing positions (with occasional dynamic changes) rather than rapidly flipping amongst them.[4] This requires a theory that can explain stable price dispersion. We need an equilibrium in pure strategies with heterogeneous price choices.

We make our "no charging over list price" assumption because it is often easy to discount a list price, but difficult to raise it. There are at least two applied motivations for this claim.

Firstly, legal constraints may force a firm to meet any published offer. In his study of dispersed prices for prescription drugs Sorensen (2002, p. 837) reported that "price-posting legislation dictates that any posted price must be honored at the request of the consumer." Charging an "over" at the point of sale can fall under many authorities' definitions of deceptive pricing. For example, the UK's Advertising Standards Authority advises that there should be minimal availability of a product at a posted price. Similar advice is provided by Ireland's Competition and Consumer Protection Division. Retailers using barcoding systems with GS1 New Zealand adopt a code of practice that prevents charging above a displayed price. Even if some price rises are allowed, there can be other limitations. Obradovits (2014) documented the Austrian

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[2]Kaplan and Menzio (2015) distinguished between three components of variation: a store-specific component, across stores for all products; a store-specific product component, from variation across stores for individual products; and a transaction component, defined as within-store variation for the same product on different days. This final component accounts for a substantial fraction (approaching one half) of variation, but not all of it. Kaplan and Menzio (2015) defined a product by its barcode and the quarter in which it is sold. Using a subset of Kilts-Niesen data, Kaplan, Menzio, Rudanko, and Trachter (2016) reported that "a sizeable portion of the cross-sectional variation in the price at which the same good trades … is due to the fact that stores that are, on average, equally expensive set persistently different prices for the same good." A related study by Eden (2018) used data from Information Resources to find price variation at the barcode-week level.

[3]If the symmetric mixed equilibrium of Varian (1980) were played each period then the variance of prices through time for a particular firm (the transaction component) would match the variance across all firm-time pairs.

[4]Many industry-specific studies that document price dispersion for products including prescription drugs (Sorensen, 2002), illicit drugs (Galenianos, Pacula, and Persico, 2012), memory chips (Moraga-González and Wildenbeest, 2008), and textbooks (Hong and Shum, 2010), are also consistent with this summary.

gasoline market: a regulation from 2011 prohibited firms from raising their prices more than daily, and restricted them to implement any price rise at noon. Price cuts were freely permitted.[5]

Secondly, customers may see any attempt to charge above a list price as unfair, may find it socially unacceptable, or it may otherwise reduce their demand. For example, if a list price sets a reference point then loss-aversion arguments (Kahneman and Tversky, 1979) may suggest a higher elasticity of demand above the list price than below (Ahrens, Pirschel, and Snower, 2017). Marketing researchers have documented how "advertised reference prices" can impact consumers' perception of value and purchase intentions (e.g., Urbany, Bearden, and Weilbaker, 1988; Lichtenstein, Burton, and Karson, 1991; Grewal, Monroe, and Krishnan, 1998; Alford and Engelland, 2000; Kan, Lichtenstein, Grant, and Janiszewski, 2013). The role of fairness concerns as a constraint on profit-seeking, was of course, central to the work of Kahneman, Knetsch, and Thaler (1986). Relating their ideas to Okun (1981), they noted "the hostile reaction of customers to price increases that are not justified by increased costs and are therefore viewed as unfair." The importance of fairness considerations in pricing is central to other marketing studies (Campbell, 1999, 2007; Bolton, Warlop, and Alba, 2003; Xia, Monroe, and Cox, 2004).

Our "no charging above list price" assumption is readily incorporated into the Varian (1980) "model of sales" setting. In that benchmark model, customers are either shoppers ("informed") who buy at the lowest price, or are captive to one firm ("uninformed" about other suppliers). Starting from any profile of high prices for the same product, some firm will undercut the lowest price and so capture the shoppers. Further best-replies walk the firms down a staircase of prices. At sufficiently low prices firms abandon the hunt for shoppers and instead exploit captive customers, which elevates prices back up to the monopoly level. This "Edgeworth cycle" logic (Maskin and Tirole, 1988a,b) rules out an equilibrium in pure strategies.[6]

If raising price is impossible, then the "elevator" from a shopper-capturing low price to a captive-exploiting high price is removed. Furthermore, the ability of a firm to post a price in advance offers commitment power. In a subgame perfect equilibrium, one (aggressive) firm posts a low list price and serves all shoppers; other firms post high list prices to exploit fully their captive customers. A sufficiently low limit price dissuades those others from discounting.

Adopting a natural two-stage pricing process (simultaneous posting of list prices, followed by the opportunity to offer discounted retail prices) we find a unique price profile that is implemented via pure strategy choices along the (subgame perfect) equilibrium path.[7]

We predict stable price dispersion, in the sense that prices vary across firms and yet the play of pure strategies eliminates any incentive to deviate. Working with asymmetric firms, we identify

[5]Obradovits (2014) reported that similar regulations were proposed in New York State and in Germany.

[6]More formally: if there is a unique lowest price then the lowest-price firm could profitably raise it; if multiple firms are cheapest then one could undercut the others. Varian (1980) constructed a symmetric equilibrium in which firms continuously randomize over an interval which extends downward from the reservation price of captive customers; Baye, Kovenock, and de Vries (1992) characterized the full set of equilibria.

[7]Another recent study of list prices and subsequent discounts was presented by Anderson, Baik, and Larson (2016). Their model generates mixed-strategy pricing, and their study focuses on the phenomenon of targeted price discrimination (discounts) and the effects of cheaper targeting technology.

the limit-pricing firm that captures the shoppers. A firm is more willing to act aggressively if it has a smaller captive customer base, and is more able to do so if it enjoys a lower marginal cost. These parameters determine the firm that can offer the lowest undominated list price, and it is this firm that captures the shoppers. Helpfully, our equilibrium shares several comparative-static properties of standard single-stage specifications, and so is not in conflict with them.

An advantage of our approach is that it predicts price dispersion in pure strategies, and yet yields expected profits that match those from a single-stage model. A disadvantage is that, in the captives-and-shoppers world, the dispersion involves only two prices. This is because there is, in effect, only a single shopper type that performs comparisons amongst all prices. A fuller specification would allow for an additional customer type for each possible "consideration set" of suppliers. To obtain a richer pattern of dispersion, to incorporate more consideration sets, and to generate deeper results, we extend our approach to a suite of three broader models. For each model, we find a unique profile of entirely distinct prices that are played as pure strategies. When firms are asymmetric, we predict which firm charges which price. For these three cases, we generate novel results on endogenous product prominence, advertising exposure, and costly buyer search.

Our first extended setting is a model of prominence. A prominent firm (perhaps a national supplier) has some captive customers, but is also available to all other customers. Non-prominent rivals (for example, local suppliers) only reach customers who also see the non-prominent firm's price. We find that each firm charges a distinct price. The prominent firm is the most expensive.[8] Others post prices that are sufficiently low to dissuade the prominent firm from undercutting them. Amongst the non-prominent firms, the one with a larger reach (and so the one that is most tempting for the prominent firm to challenge) is cheaper.

Our model of pricing prominence also provides an opportunity for us to demonstrate how the two-stage pricing framework can be a component of a deeper model. We add an earlier stage to consider the incentives of a "prominence provider" who elevates one of many local firms to national prominence. We find that this provider makes a prominence offer (which is accepted) to the local firm with the largest reach (equivalently, the largest local customer base).

Our second extended setting builds upon classic models of informative advertising (Butters, 1977; Grossman and Shapiro, 1984): buyers are randomly and independently aware of each firm. The reach of a firm corresponds to the fraction of buyers who see its price. This specification generates the full set of buyer consideration sets: captive consumers, those who see two prices, shoppers who see all prices, and so on. Adopting our two-stage list-prices-then-discounts approach, we find that entirely distinct prices are posted (and not discounted) using pure strategies. Firms with broader advertising exposure charge higher prices.

We endogenize costly advertising decisions via a three-stage model: firms choose advertising (the breadth of awareness), followed by our pricing game; this adds a list-price stage and positive

[8]This is also true in the search-and-prominence duopoly model of Moraga-González, Sándor, and Wildenbeest (2018). For contrasting results in a sequential search model, see Armstrong, Vickers, and Zhou (2009).

advertising costs to the model of Ireland (1993). A distinct largest-spend firm emerges, while rivals employ "puppy-dog ploy" strategies: each (endogenously smaller) firm advertises to at most half of potential buyers. By limiting its exposure, a firm makes it less tempting for a higher-priced competitor to undercut, which allows that firm to list a higher price. Under the "random mailbox postings" technology of Butters (1977), the leading firm advertises at exactly twice the intensity of all its rivals; and as advertising costs fall this leading firm reaches out to everyone. We predict a distinct pattern with one (near) universally known supplier (perhaps a direct sales channel by the product manufacturer) charging the highest feasible price, and a set of progressively cheaper retailers, each known to only a minority of customers.

Our first two extensions allow the price-awareness of buyers to depend on the decisions of firms. In our third and final extended setting, awareness is chosen by buyers. We build upon the fixed-sample search technology of Burdett and Judd (1983) and Janssen and Moraga-González (2004): buyers choose how many (costly) price quotations to request and then select the best available price. Firms set prices using our two-stage approach. We predict that firms choose a distinct set of prices. Search behavior and the comparative-static properties differ from those of Janssen and Moraga-González (2004): we find (for reasonable parameters) that buyers obtain either one or two quotations, and that the expected price charged falls with entry.

Our unique equilibrium prediction for prices stands in contrast to the single-stage paradigm. Established models often feature a symmetric distribution of consumer types across firms and derive a symmetric equilibrium. However, with symmetrically-distributed consumers it is known that vast multiplicities of equilibria can exist (Baye, Kovenock, and de Vries, 1992), and relatively little is known about equilibrium under asymmetric assumptions.[9] In all the settings we study, we predict a unique profile of prices under either symmetric or asymmetric specifications.

In summary, our two-stage approach predicts stable price dispersion; it is easily used as a replacement for the clearinghouse stage of a richer model; and it generates powerful new insights into prominence provision, informative advertising, and costly buyer search.

Section 2 studies our two-stage model of sales à la Varian (1980), and presents new comparative-static results. Sections 3 to 5 develop the aforementioned extensions to include product prominence, informative advertising, and costly buyer search. Section 6 highlights the broad applicability of our approach by citing a range studies that use a clearinghouse pricing stage.

[9]Some exceptions include the duopoly of Narasimhan (1988); the triopoly of Armstrong and Vickers (2018b); and (in their Section V) Baye, Kovenock, and de Vries's extension of Varian (1980).

2. A Model of Sales

Here we modify the classic model of sales to allow for two stages of pricing: the posting of list prices followed by the opportunity to offer discounts. A main result (Proposition 1) reports a unique set of prices that are chosen as pure strategies along the equilibrium path.

A Two-Stage Pricing Game. The game is played by n firms across two stages of play. At the first stage firms simultaneously post list prices, where $\bar{p}_i$ is the list price of firm $i \in \{1, 2, \ldots, n\}$. These prices are observed by all firms. At the second stage the firms simultaneously set final retail prices. The final price p_i of each firm i must satisfy $p_i \leq \bar{p}_i$. A choice $p_i < \bar{p}_i$ means that firm i offers a discount. Firm i faces a constant marginal cost $c_i \geq 0$.

Buyers are all willing to pay v, which exceeds the marginal cost of every firm. A mass $\lambda_i > 0$ of buyers are "loyal" or "captive" to firm i. A mass $\lambda_S > 0$ of "shoppers" buy from the cheapest firm. If there is lowest-price tie then we assume (for convenience) that the shoppers buy from the firm (or firms) with the lowest marginal cost.[10] Ignoring tied prices for now, the profit (and payoff) of firm i is $(p_i - c_i)\,(\lambda_i + \lambda_S \mathfrak{I}[p_i < p_j\,\forall j \neq i])$, where "$\mathfrak{I}[\cdot]$" is the indicator function.

In the absence of constraining first-stage list prices (if, for example, $\bar{p}_i = v$ for all i) the second stage becomes the setting of Varian (1980, 1981), generalized to allow for asymmetries in costs and loyalties. Any equilibrium of a single-stage symmetric game involves mixing by at least two firms, and generates profits equal to those earned by serving only captive customers at the maximum price v (Baye, Kovenock, and de Vries, 1992). In contrast, we seek an equilibrium (of our two stage game) in which firms choose pure strategies. (We will show that in such an equilibrium the second-stage prices equal the first-stage list prices; no discounts are offered.)

Definition. *A profile of prices generates an equilibrium in pure strategies if there is a subgame perfect equilibrium in which those prices are chosen as pure strategies on the equilibrium path.*

We do not insist that pure strategies are used off the equilibrium path.[11] (In fact, our equilibria involve the play of mixed equilibria in some off-equilibrium-path subgames.)

A Unique Equilibrium Outcome in Pure Strategies. In the Varian (1980) model, a firm captures shoppers by undercutting the lowest competing price. This force walks the firms down a staircase of prices, until prices are so low that firms elevate prices upward to exploit loyal customers. The undercutting process then resumes; there is no pure-strategy equilibrium.

In our model a first-stage price commits a firm to a second-stage price cap. If the cap is set sufficiently low then a firm can ensure that its competitors have no wish to undercut in the second stage. The equilibrium structure, then, involves all but one firm pricing high while the remaining firm posts a low limit price to dissuade any second-stage discounts.

[10] This tie-break rule easily establishes the existence of Nash equilibria in all possible subgames.

[11] We can also work with a weaker solution concept: we do not require equilibria in subgames that are reached following a multi-player deviation or following a single-player deviation that is equilibrium-dominated.

Turning to our formal analysis, firm i guarantees a profit of at least $\lambda_i(v - c_i)$ by setting $p_i = \bar{p}_i = v$ and exploiting captive customers, even if shoppers go elsewhere. A strictly lower price can at best win the business of all shoppers and generate a profit of at most $(\lambda_i + \lambda_S)(p_i - c_i)$. It follows that firm i will never set a list price (and final sales price) where this falls below $\lambda_i(v - c_i)$: list prices $\bar{p}_i < p_i^\dagger$ are strictly dominated, where

$$p_i^\dagger \equiv \frac{\lambda_i v + \lambda_S c_i}{\lambda_i + \lambda_S}. \tag{1}$$

Eliminating strictly dominated prices, firm i chooses $\bar{p}_i \in [p_i^\dagger, v]$. The minimum undominated price $p_i^\dagger$ is increasing in c_i and λ_i. If these firm-specific parameters are lower then a firm can be more aggressive, in the sense that it is more willing (lower λ_i) and able (lower c_i) to compete for shoppers' business. To reflect this, we label the firms in order of increasing aggressiveness. The statements of our results are simplified if this order is strict: $p_1^\dagger > \cdots > p_n^\dagger$.

Definition. *The competing firms are* strictly asymmetric *if their minimum undominated prices are strictly ordered:* $p_1^\dagger > \cdots > p_n^\dagger$. *A higher-indexed firm is described as* more aggressive.

Henceforth we often assume (with very little loss of generality) that the firms are strictly asymmetric. We will also discuss situations in which firms are exactly symmetric, and our results extend appropriately to other situations in which subsets of firms are the same.[12]

Any pure-strategy equilibrium of a second-stage subgame satisfies two properties. Firstly, the cheapest firm must be unique: if two (or more) firms tie for the lowest price, then one faces an incentive to undercut. Secondly, all firms charge their list prices: given that the lowest-price firm is unique, it has no incentive to discount; a higher-priced firm sells only to captive customers, and so can profit by raising any discounted price all the way up to its list price.

It follows that any equilibrium with on-path pure strategies involves a unique lowest list price. This cheapest firm subsequently captures all shoppers. Given that higher-price firms sell only to their captive customers, their list prices must be as high as possible. We conclude that some firm j charges a list price $\bar{p}_j < v$, whereas all other firms $i \neq j$ set $\bar{p}_i = v$.

In the second stage no firm is able to raise price. The lowest-price firm has no incentive to lower its price. We must check, however, whether one of the high-price firms wishes to offer a second-stage discount and undercut $\bar{p}_j$. Doing so is unprofitable if and only if $\bar{p}_j \leq p_i^\dagger$. We conclude that $\bar{p}_j \leq \min_{i \neq j} p_i^\dagger$. If this inequality holds strictly then firm j could safely raise its price at the first stage, hence $\bar{p}_j = \min_{i \neq j} p_i^\dagger$. For firm j, any list price below $p_j^\dagger$ is strictly dominated, and so $p_j^\dagger \leq \bar{p}_j = \min_{i \neq j} p_i^\dagger$. This implies that the lowest-price is offered by the most aggressive firm. Lemma 1 summarizes.[13]

[12] Accommodating fully those other situations (in which firms are not strictly asymmetric) lengthens and complicates the statements of our results without generating any new insights.
[13] Versions of this lemma hold when firms are not strictly asymmetric. For example, if firms are symmetric then any one of them can play the role of the lowest-list-price competitor that captures all of the shoppers.

Lemma 1 (Necessary Properties of Prices). *If a profile of prices generates an equilibrium in pure strategies then the most aggressive firm sets* $\bar{p}_n = p^\dagger_{n-1}$ *and captures shoppers, while less aggressive firms* $i < n$ *set* $\bar{p}_i = v$ *and sell only to captive customers. No firm offers a discount.*

Lemma 1 characterizes the (unique) profile of list prices that can form the equilibrium play of pure strategies. However, it does not establish that such an equilibrium exists.

Consider a strategy profile that satisfies Lemma 1: $\bar{p}_n = p^\dagger_{n-1}$ and $\bar{p}_i = v$ for $i < n$. For the high-price firms, the only possible first-stage deviation is a price cut. Given that firm n is guaranteed to price at or below $p^\dagger_{n-1}$, any downward deviation by $i < n$ leaves firm i strictly worse off. Similarly, firm n serves all shoppers and cannot gain by deviating downward.

The remaining first-stage deviation is an increase in $\bar{p}_n$. This leads to a subgame in which there is no pure-strategy Nash equilibrium. The next lemma characterizes the profits of firms in any mixed-strategy equilibrium of such a subgame.

Lemma 2 (Second-Stage Subgame). *Consider a subgame in which firm n faces a price cap satisfying* $v \geq \bar{p}_n > p^\dagger_{n-1}$ *and others are unconstrained, so that* $\bar{p}_i = v$ *for all* $i < n$*. There is at least one mixed-strategy Nash equilibrium of this subgame. In any such equilibrium, firm n earns expected profit* $(\lambda_S + \lambda_n)(p^\dagger_{n-1} - c_n)$ *while each firm* $i < n$ *earns expected profit* $\lambda_i(v - c_i)$*.*

For example, if firm n deviates to a list price satisfying $p^\dagger_{n-2} > \bar{p}_n > p^\dagger_{n-1}$ then in the subgame there is a unique Nash equilibrium: firms $i < n-1$ maintain their list prices ($p_i = \bar{p}_i = v$), while firms $n-1$ and n mix continuously over the interval $[p^\dagger_{n-1}, \bar{p}_n)$ with distribution functions

$$F_{n-1}(p) = 1 - \frac{\lambda_S(p^\dagger_{n-1} - c_n) - \lambda_n(p - p^\dagger_{n-1})}{\lambda_S(p - c_n)} \quad \text{and} \quad F_n(p) = 1 - \frac{\lambda_{n-1}}{\lambda_S}\frac{v - p}{p - c_{n-1}}, \tag{2}$$

and place remaining mass at $\bar{p}_{n-1}$ and $\bar{p}_n = v$, respectively. In fact, under some relatively weak parameter restrictions this is an equilibrium for any $\bar{p}_n > p^\dagger_{n-1}$.[14]

The profits reported in Lemma 2 are the same as those earned when firms use the list prices described in Lemma 1. This means that an upward deviation in list price by firm n is not profitable. Finally, within (off-path) subgames following any other choices of list prices, any equilibrium may be played. Together, this leads us to our first main result.

Proposition 1 (Pure Strategies on the Equilibrium Path). *There is a unique profile of prices that generates an equilibrium in pure strategies. There are no second-stage discounts. The most aggressive firm sets a price* $\bar{p}_n = p^\dagger_{n-1}$ *that dissuades others from undercutting, and serves all shoppers. Others sell only to captive customers at the maximum possible price.*

Recall that firms are strictly asymmetric. If firms are symmetric then there are n price profiles that generate an equilibrium with the play of pure strategies. These differ only via the identity of the aggressive firm that sets the low price and captures the shoppers.

[14] For example, if firms share a common marginal cost then this is an equilibrium for any $\bar{p}_n > p^\dagger_{n-1}$.

Profits and Welfare. The strategies of Proposition 1 yield a profit of $\lambda_i(v - c_i)$ to each firm $i < n$ (the monopoly profit from serving only captive buyers) and a profit of $(\lambda_S + \lambda_n)(p^\dagger_{n-1} - c_n)$ for the aggressive firm n. From Lemma 2, these profits equal those from the mixed-strategy equilibria of a pricing game when firm n deviates to a higher list price $\bar{p}_n > p^\dagger_{n-1}$. One such deviation is to $\bar{p}_n = v$. Following this deviation, no firm is constrained in the second stage, and so the subgame is equivalent to the single-stage pricing game studied (for symmetric firms) by Varian (1980) and (more generally) by Baye, Kovenock, and de Vries (1992).

Corollary 1 (Profits). *The unique profile of prices that generates an equilibrium in pure strategies yields profits equal to those earned in a standard single-stage model of sales.*

This implies that a Varian (1980) pricing stage of any model can be replaced with our two-stage list-prices-then-discounts version without upsetting early-stage behavior of profit-seeking firms.

The switch to two-stage pricing does not influence profits, but it can matter for welfare. This is because output is allocated differently. In our model, shoppers are served by a single firm. In contrast, a mixed-strategy equilibrium allocates shoppers randomly. If costs are heterogeneous and if the most aggressive firm has the lowest marginal cost then our equilibrium is more efficient. Of course, if the most aggressive firm achieves that status because of a small mass of captive customers, then the list-prices-and-discounts scenario can result in lower welfare.

Corollary 2 (Welfare and Consumer Surplus). *If the asymmetry of firms is driven by heterogeneity of costs, so that the most aggressive firm has the lowest marginal cost, then an equilibrium with on-path pure strategies in a two-stage model generates higher welfare and higher consumer surplus than in any equilibrium of a single-stage pricing game.*

Finally, we comment briefly on pricing patterns. The symmetric equilibrium of Varian (1980) generates generically distinct prices. Baye, Kovenock, and de Vries (1992) showed that all (of the uncountably-many) equilibria require at least "two to tango" by randomizing over the range of undominated prices.[15] In our equilibrium "one firm dances alone" and so the extent of price dispersion consists of only two distinct prices. We return to discuss this feature toward the end of this section.

Comparative-Static Results. The effect of the composition of consumers and the number of firms has been of interest to many authors (Varian, 1980; Rosenthal, 1980; Stahl, 1989; Janssen and Moraga-González, 2004; Morgan, Orzen, and Sefton, 2006; Armstrong, 2015). Most restricted attention to a symmetric industry, and so set (without loss) marginal cost to zero. We do this here too.

There are alternative assumptions about how the number of captive consumers depends on entry: either a fixed mass are divided evenly between firms (Varian, 1980) or new entrants

[15]Using a single-stage symmetric model, Johnen and Ronayne (2019) show that there is a unique and completely mixed equilibrium if and only if there are some buyers who see exactly two prices.

bring their own loyal customers (Rosenthal, 1980). We use the former specification: $\lambda_i = \lambda_L/n$ for some total mass λ_L of captive buyers. (Later we discuss the latter specification.)

Under symmetry, we pick the low-price supplier to be firm n and we consider prices that generates an equilibrium in pure strategies. Applying Proposition 1,

$$\bar{p}_i = v \text{ for } i < n \quad \text{and} \quad \bar{p}_n = p^\dagger = \frac{\lambda_L v}{\lambda_L + n\lambda_S}. \tag{3}$$

The shopper-capturing lowest price $p^\dagger$ combines the two forces highlighted by Janssen and Moraga-González (2004). The *business stealing* incentive to set a low price to capture shoppers is increasing in λ_S, while the *surplus appropriation* incentive to exploit captives with a high price is increasing in λ_L/n. The shoppers exert a search externality (in the spirit of Armstrong, 2015) by driving down the price paid by the captive customers of the low-priced firm n.[16]

Entry to the industry reduces the lowest price, which is paid by shoppers and by the mass λ_L/n of customers who are loyal to firm n. However, it also re-allocates the captive buyers toward the other high-priced firms. The average price can be readily calculated in two ways

$$\frac{\sum_{i=1}^{n} \bar{p}_i}{n} = \frac{(\lambda_L + (n-1)\lambda_S)v}{\lambda_L + n\lambda_S} \quad \text{and} \quad \frac{\sum_{i=1}^{n-1} (\lambda_L/n)\bar{p}_i + [\lambda_S + (\lambda_L/n)]\bar{p}_n}{\lambda_L + \lambda_S} = \frac{\lambda_L v}{\lambda_S + \lambda_L}. \tag{4}$$

The first expression is the simple average of prices across firms. It is also the average price paid by a captive buyer. The second expression (an appropriately weighted average) is the average price paid overall. These expressions generate a further corollary to Proposition 1.

Corollary 3 (Consumer Composition and Entry to a Symmetric Industry). *Consider the unique profile of prices that generates an equilibrium in pure strategies for a zero-cost industry in which a mass λ_L of captive customers is divided equally amongst n firms. The average price (both charged and paid) rises with λ_L, but falls with λ_S. Entry to the industry raises the average price charged by suppliers, but the does not influence the average price paid by buyers.*

The final claim (the average price paid is independent of n) holds because symmetric firms earn the same profit as they would from selling only to their captive customers at the maximal price v. This means that industry profit equals $\lambda_L v$. Changing n while keeping both λ_L and λ_S fixed does not change industry profit and so does not change the average price paid.

This logic also applies if, following Rosenthal (1980), an entrant brings new loyal customers. We set $\lambda_i = \lambda_L$ for all i, so that the total mass of captive buyers is $n\lambda_L$, and the total mass overall is $\lambda_S + n\lambda_L$. Each firm earns a profit equal to that earned by charging v to captives. Industry profit is $n\lambda_L v$ which increases with entry. The average price paid by buyers is (as an accounting identity) equal to the average profit per consumer, $n\lambda_L v/(\lambda_S + n\lambda_L)$. By inspection, entry raises the average price paid. Entry itself has no direct effect on industry profitability; in essence suppliers earn monopoly profits on captives and compete away profits (for any n) corresponding to shoppers. What matters for profitability is the balance between captives

[16] In the symmetric equilibrium of a single-stage model, they lower the expected price paid by all loyal customers.

and shoppers. The specification of Rosenthal (1980) links that balance to the entry of new competitors, generating the result that entry raises the average price paid.

Of course, the results here are also found in single-stage models of sales (where each firm also earns a profit equal to that gained from focusing on the exploitation of captive customers) and so, under symmetry, we add support for established comparative-static predictions.

The profitability of an asymmetric industry can depend on the distribution of captives across the n firms. Each of the $n-1$ largest firms ($i < n$) earns profit $\lambda_i v$. The most aggressive firm (with the smallest loyal customer base) serves the shoppers at the price $p^{\dagger}_{n-1}$. So

$$\text{profit of firm } n = (\lambda_S + \lambda_n) p^{\dagger}_{n-1} = \frac{(\lambda_S + \lambda_n)\lambda_{n-1} v}{\lambda_{n-1} + \lambda_S} = \lambda_n v + \frac{\lambda_S(\lambda_{n-1} - \lambda_n) v}{\lambda_{n-1} + \lambda_S}. \tag{5}$$

Adding in the profits for the first $n-1$ firms, and dividing by the total mass of customers $\lambda_L + \lambda_S$ yields the average price paid or (equivalently) the industry's profitability. This is

$$\text{average price paid} = \frac{\text{industry profit}}{\lambda_L + \lambda_S} = \frac{\lambda_L v}{\lambda_L + \lambda_S}\left(1 + \frac{\lambda_S}{\lambda_L}\frac{\lambda_{n-1} - \lambda_n}{\lambda_{n-1} + \lambda_S}\right). \tag{6}$$

This average price is higher in an asymmetric industry than in a symmetric industry with the same captives-to-shoppers ratio. The effect is determined by the ratio $(\lambda_{n-1} - \lambda_n)/(\lambda_{n-1} + \lambda_S)$, and requires a gap in the sizes of the two smallest firms. Profitability is larger when the smallest firm is smaller (fewer captives are able to buy at the aggressive low price) and when the second-smallest firm is larger (this firm is less tempted to undercut firm n, and so firm n can select a higher limit price). Industry profitability is maximized by moving captive customers away from the smallest firm, away from the largest $n-2$ firms, and toward the second-smallest firm. The best configuration (for profitability) is for firm n to have no captive customers, and for the remaining firms to share them equally: $\lambda_n = 0$ and $\lambda_i = \lambda_L/(n-1)$ for $i < n$.

Based on these observations, it is easy to see that adding an extra competitor to the industry (while keeping fixed the mass of captive buyers, and so redistributing some to the new entrant) can either raise or lower the average price paid. Beginning from a symmetric industry configuration, adding a smaller new firm (and so creating a gap between the smallest two firms) raises the average price. Adding a further firm with an identical size to the existing smallest firm then lowers the average price back to its original level.

The Extent of Price Dispersion. We predict stable price dispersion, in the sense that firms play pure strategies and different prices are chosen. However, the extent of dispersion is small: only two prices are offered, and $n-1$ firms coalesce on one of those prices. The reason is that the simplest captives-and-shoppers specification features shoppers as the only customer type engaging in price comparisons. Given shoppers see every price, there is only room for one single low-priced firm, who caters for that type while others focus on captive customers.

Over the next three sections we consider an expanded suite of models which allow for richer consumer consideration sets and predict n distinct prices in equilibrium.

3. A Model of Prominence

Moving beyond the captives-and-shoppers setting, we allow one prominent firm to be visible to everyone. Buyers also see at most one other firm. The prominent firm might correspond to a national sales channel, whereas other firms might be local suppliers. This simple specification generates entirely distinct prices.[17] Moreover, it can be incorporated into a deeper model: we add a preliminary stage in which a prominence provider sells the prominence position.

A Triopoly with a Prominent Firm. Firm $i \in \{1,2,3\}$ has access to a base of ϕ_i customers who each see firm i's final retail price. Additionally, all potential customers are informed of one, prominent, firm's price. Let that be firm $i = 1$. Potential customers are partitioned into three types with consideration sets $\{1\}$, $\{1,2\}$, and $\{1,3\}$. Only the first type is truly captive; the others compare the price of their "local" supplier $i \in \{2,3\}$ to that of the prominent firm. We set symmetric marginal costs to zero, and we label the non-prominent firms so that $\phi_2 \geq \phi_3$. (For symmetric firms we also report a solution for an oligopoly with $n > 3$ competitors.)

Other aspects of our model are as before: firms post list prices in a first stage, and are free to discount (but not raise) those prices in a second stage. We seek prices that are chosen as pure strategies along the equilibrium path of a subgame perfect equilibrium.

Equilibrium Prices. The prominent firm charges the maximum possible price and sells only to captives, while the non-prominent firms charge strictly less. The reason is that a non-prominent firm has no purely captive customers, and so always undercuts the (strictly positive, because it has captive customers and so is strictly profitable) price of the prominent firm. Given that the prominent firm loses comparison sales to its non-prominent competitors, it is optimal to exploit fully the captive customers who only see the prominent price, by setting $p_1 = \bar{p}_1 = v$.

A non-prominent firm sets a list price just low enough to dissuade the prominent firm from undercutting it. Suppose that $j \in \{2,3\}$ offers the lowest list price. The prominent firm earns $\phi_1 v$ by exploiting captives, but captures everyone, a mass $\phi_1 + \phi_2 + \phi_3$, by undercutting $\bar{p}_j$. It follows that the critical list price satisfies $(\phi_1 + \phi_2 + \phi_3)\bar{p}_j = \phi_1 v$. The solution for $\bar{p}_j$ does not dependent on which non-prominent firm $j \in \{2,3\}$ posts this lowest list price.

Next, consider the other non-prominent firm $i \neq j$. It can lift its list price above $\bar{p}_j$ (an undercut of $\bar{p}_i > \bar{p}_j$ by the prominent firm would only capture $\phi_1 + \phi_i$ customers, and so is less tempting if $\bar{p}_i$ is not too high) and so $\bar{p}_i > \bar{p}_j$. The maximum list price which prevents the undercut satisfies $(\phi_1 + \phi_i)\bar{p}_i = \phi_1 v$. The solution for this intermediate list price $\bar{p}_i$ depends on the identity of the firm $i \in \{2,3\}$ that offers it; it is higher if ϕ_i is smaller.

[17] In ongoing work (Myatt and Ronayne, 2019) we study nested prominence: the audience of any firm is contained within that of any more prominent supplier. (We think here of an ordered list of search results, where a buyer of type i examines only the first i listings.) In this setting, we again characterize a unique profile of entirely distinct prices supported by the play of pure strategies. For related single-stage models, see Inderst (2002) and Armstrong and Vickers (2018b).

This arguments pin down two candidates for a profile of prices. These two profiles are

$$\bar{p}_1 = v, \quad \bar{p}_i = \frac{\phi_1 v}{\phi_1 + \phi_i}, \quad \text{and} \quad \bar{p}_j = \frac{\phi_1 v}{\phi_1 + \phi_2 + \phi_3} \quad \text{for} \quad i, j \in \{2, 3\} \quad \text{and} \quad j \neq i, \tag{7}$$

with prices that satisfy $\bar{p}_1 > \bar{p}_i > \bar{p}_j$. If the non-prominent firms are symmetric (so that $\phi_2 = \phi_3$) then there is a unique profile of prices, although the two firms can take alternative positions in that profile. If $\phi_2 > \phi_3$ then the profiles strictly differ.

From either set of retail prices, no firm can profitably deviate downwards: non-prominent firms have nothing to gain and the construction of those list prices removes the incentive for the prominent firm to undercut. To construct a subgame perfect equilibrium, we need to check whether any non-prominent firm wishes to deviate to a higher list price.

In the proof of Proposition 2 we show that we can do this so long as the larger non-prominent firm chooses the lowest list price, so that the price profile satisfies $\bar{p}_1 > \bar{p}_3 > \bar{p}_2$. For such a profile we prove a version of Lemma 2: an upward deviation by either non-prominent firm results in no gain in any equilibrium of the relevant subgame. However, there is a profitable upward deviation from the list-price profile where $\bar{p}_1 > \bar{p}_2 > \bar{p}_3$.[18] This means that (if firms are asymmetric) we are able to pin down a unique profile of distinct list prices.

If firms are symmetric then our arguments so far already pin down a unique profile, although they do not establish the order of the firms in the pricing sequence. For symmetric firms we also report the unique profile for a larger oligopoly with $n > 3$ firms.

Proposition 2 (Pure Strategies in a Model of Prominence). *If the non-prominent firms are asymmetric, so that $\phi_2 > \phi_3$, then the unique profile of distinct prices*

$$\bar{p}_1 = v, \quad \bar{p}_3 = \frac{\phi_1 v}{\phi_1 + \phi_3}, \quad \textit{and} \quad \bar{p}_2 = \frac{\phi_1 v}{\phi_1 + \phi_2 + \phi_3} \tag{8}$$

generates an equilibrium in pure strategies. There are no second stage discounts. The prominent firm sets the monopoly price. The larger non-prominent firm sets the lowest price.

If firms are symmetric, so that $\phi_i = \phi$ for all $i \in \{1, \ldots, n\}$ for general n, then there is a unique profile of distinct prices which generates an equilibrium in pure strategies:

$$\bar{p}_i = \frac{v}{i} \quad \textit{for all} \quad i \in \{1, \ldots, n\}, \tag{9}$$

so that the list (and final retail) price of a firm declines inversely to its position in the sequence.

Unlike the model of sales (in Section 2) and our model of advertising (in Section 4) firms with larger reach can be cheaper: a larger reach makes a non-prominent firm's price more attractive to undercut, pushing down that firm's price. It remains the case that the profit of a non-prominent firm is increasing in its own size. However, the larger non-prominent firm can easily make a smaller profit than the other. This is true whenever their sizes are sufficiently close.

[18] Specifically, the smallest firm $i = 3$ has an incentive to deviate to a list price strictly above $\bar{p}_2$ in this case.

As the prominent firm's position strengthens (an increase in ϕ_1) then price cuts (which sacrifice revenue from captives) hurt it more. As a result, its non-prominent rivals can sustain higher prices without being undercut. It follows that non-prominent firms' prices and profits are increasing in ϕ_1. This implies that consumers are worse off with a larger prominent firm.

Selling Prominence. The prominent firm is advantaged relative to others. We now extend our model to an environment in which this advantage is conferred by a prominence provider.

Specifically, suppose that all firms in the triopoly begin with exclusive local customer bases, so that firm $i \in \{1, 2, 3\}$ charges v to ϕ_i buyers within its locality. A unique (monopolist, labeled as M) prominence provider offers, in an initial stage, to elevate one firm to national prominence. For example, this provider may be a department store which chooses which product to display in the window, or a website that decides which product to put on its home page.

Specifically, M makes a take it or leave it offer to one firm, and promises to give prominence to a specified competitor if the offer is refused. We label the firms so that $\phi_1 > \phi_2 > \phi_3$.

Consider an offer to the largest firm. This firm earns $\phi_1 v$ from a prominent position. If it loses prominence then it will (Proposition 2) take the lowest-price position and so earn $\phi_1\phi_i v/(\phi_1 + \phi_2 + \phi_3)$ where $i \in \{2, 3\}$ is the firm that rises to prominence otherwise. Clearly, the harshest threat is for the provider to declare that $i = 3$. The provider can make an offer

$$\text{prominence fee} = \phi_1 v - \frac{\phi_1\phi_3 v}{\phi_1 + \phi_2 + \phi_3} = \frac{\phi_1(\phi_1 + \phi_2)v}{\phi_1 + \phi_2 + \phi_3} \tag{10}$$

to the largest firm, and this offer will be accepted.

Of course, the prominence provider could make an offer to a smaller firm. The proof of Proposition 3 below proves that this generates a lower fee than dealing with the largest firm.

Proposition 3 (Prominence Provision). *If firms play pure strategies in the two price-setting stages, then there is a unique outcome from a subgame perfect equilibrium in which (i) the prominence provider offers prominence to the largest firm in exchange for a fee equal to $\pi_M = \phi_1(\phi_1 + \phi_2)v/(\phi_1 + \phi_2 + \phi_3)$; (ii) this firm accepts; and (iii) firms post and do not discount the prices reported in Proposition 2. The prominence fee satisfies*

$$\frac{\partial \pi_M}{\partial \phi_1} > \frac{\partial \pi_M}{\partial \phi_2} > 0 > \frac{\partial \pi_M}{\partial \phi_3}, \tag{11}$$

and so is largest when customers move from away from the smallest firm.

The final claim holds because non-prominent firms do worse when the prominent firm is smaller; a small prominent firm has fewer captive customers to exploit and so is more willing to undercut any non-prominent competitors. In essence, a small non-prominent rival has a threatening lean and hungry look, which strengthens the ability to extract a fee from a large firm.

4. A Model of Advertising

Our two-stage model of sales predicts stable dispersion across only two prices. A richer ladder of prices requires multiple types of shoppers. One possibility is the model of prominence from the previous section. Here, we expand the "consideration sets" for potential buyers via an advertising specification that builds upon Butters (1977), Grossman and Shapiro (1984), Ireland (1993), Chioveanu (2008), and Eaton, MacDonald, and Meriluoto (2010): each price is exposed to an independent fraction of potential buyers.

We find that a unique profile of distinct prices is supported by the equilibrium play of pure strategies. We also evaluate the endogenous choice of advertising strategies: a single firm chooses relatively high exposure for a high list price, while other firms limit their exposures in order to prevent the use of revenue-eroding lower list prices.

A Two-Stage Pricing Game with Independent Advertising. The game played by n firms is as before: firms post list prices, and then follow by setting weakly lower final sale prices. To simplify exposition we assume symmetry of costs, and then simplify again (without further loss of generality) by setting marginal cost to zero for everyone.

On the demand side, an independent fraction α_i of buyers is aware of firm i. Hence the proportion of buyers aware of firms i and j but no others is $\lambda_{\{i,j\}} = \alpha_i \alpha_j \prod_{k \notin \{i,j\}} (1 - \alpha_k)$, where the notation "$\lambda_{\{i,j\}}$" should be clear. Recycling earlier notation, the mass of shoppers is $\lambda_S = \prod_{i=1}^{n} \alpha_i$, and the mass of buyers who are captive to firm i is $\lambda_i = \alpha_i \prod_{j \neq i} (1 - \alpha_j)$.

Definition. *The firms are strictly asymmetric if* $1 \geq \alpha_1 > \cdots > \alpha_n > 0$. *A lower-indexed firm, with greater pricing awareness or advertising reach, is described as a larger firm.*

Just as before, we can accommodate ties (so that subsets of firms share the same type) but our results can be stated much more cleanly and clearly when firms are distinct.

A Unique Equilibrium Outcome in Pure Strategies. We again seek a unique outcome in pure strategies. Necessarily, this involves a pure-strategy Nash equilibrium in the pricing subgame. In the captives-and-shoppers model we noted that the lowest-price firm must be unique. Here we say something stronger: all final retail prices must be unique. If any subset of firms post the same price then there is positive probability that a buyer sees precisely this "consideration set" of suppliers and so every member of that set has an incentive to undercut.

Given that prices are distinct, any otherwise-unconstrained firm could locally raise price without losing sales. This implies that all firms must be constrained: in the second stage all firms maintain their list prices, and so no discounts are offered.

We now characterize the necessary properties of first-stage list prices. For now, suppose that $\bar{p}_1 > \cdots > \bar{p}_n$ so that larger firms list higher prices. (We will confirm that this is the case.)

The largest firm only sells to its own captive customers, and so sets the maximum list price: $\bar{p}_1 = v$. The next price must be low enough to dissuade the larger firm from undercutting in the second stage. That largest firm sells to a fraction $\alpha_1 \prod_{i>1}(1-\alpha_i)$ of buyers: those who are aware of the price p_1 but unaware of the $n-1$ cheaper firms. In contrast, charging slightly below p_2 means that only the $n-2$ cheapest firms provide competition; the undercut sells to a fraction $\alpha_1 \prod_{i>2}(1-\alpha_i)$ of buyers. There is no incentive to undercut the next list price if

$$\bar{p}_1 \alpha_1 \prod_{i>1}(1-\alpha_i) \geq \bar{p}_2 \alpha_1 \prod_{i>2}(1-\alpha_i) \quad \Leftrightarrow \quad \bar{p}_2 \leq (1-\alpha_2)\bar{p}_1. \tag{12}$$

If this "no undercutting" inequality holds strictly then the second firm can locally raise $\bar{p}_2$ without any loss of sales, and so we conclude that $\bar{p}_2 = (1-\alpha_2)\bar{p}_1$. It remains, however, to check whether the second firm wishes to raise $\bar{p}_2$ still further. One possibility would be to match $\bar{p}_1$. The worst-case scenario is that this higher price sells only to captives of this second firm. Assuming this worst-case scenario, such an upward deviation must be unprofitable, and so

$$\bar{p}_2 \alpha_2 \prod_{i>2}(1-\alpha_i) \geq \bar{p}_1 \alpha_2 (1-\alpha_1) \prod_{i>2}(1-\alpha_i) \quad \Leftrightarrow \quad \bar{p}_2 \geq \bar{p}_1(1-\alpha_1) \quad \Leftrightarrow \quad 1-\alpha_2 \geq 1-\alpha_1, \tag{13}$$

which holds if $\alpha_1 \geq \alpha_2$. This shows that the higher price must be charged by the larger firm.

We have dealt here with the two highest list prices. The same logic applies as we move down the sequence, which allows us to characterize fully the list prices that must be charged if pure strategies are played along the equilibrium path. Lemma 3 summarizes.

Lemma 3 (Necessary Properties of Advertised Prices). *If a profile of list prices generates an equilibrium in pure strategies then* $\bar{p}_1 = v$ *and* $\bar{p}_i = (1-\alpha_i)\bar{p}_{i-1}$ *for* $i > 1$.

We proceed as before. Consider any strategy profile in which the firms charge list prices which satisfy Lemma 3. By construction, and on the equilibrium path, no firm offers a discount in the second stage. In the first stage, no firm wishes to deviate with a lower list price: such a firm could (in any case) choose such a lower list price in the second stage, while lowering the first-stage list price can only serve to push down the prices of competing firms. It follows that the only possible profitable first-stage deviation is for a firm to raises its list price.

The largest firm already charges the maximum price $\bar{p}_1 = v$. For other firms, an increased list price necessarily violates the "no undercutting" constraint in the relevant subgame, which is resolved by the play of a mixed-strategy Nash equilibrium in that subgame. Such an equilibrium, however, generates payoffs that offer no improvement over the equilibrium path.

Consider, for example, an upward deviation by the second-largest firm to $\bar{p}_2 > (1-\alpha_2)v$. In the second-stage subgame, there is an equilibrium in which all other prices are maintained (so that $p_i = \bar{p}_i$ for $i > 2$) while the top two firms continuously mix over $[(1-\alpha_2)v, \bar{p}_2)$ with distributions

$$F_1(p) = \frac{1}{\alpha_1}\left[1 - \frac{(1-\alpha_2)v}{p}\right] \quad \text{and} \quad F_2(p) = \frac{1}{\alpha_2}\left[1 - \frac{(1-\alpha_2)v}{p}\right] \tag{14}$$

within that interval, with all remaining mass placed at v and $\bar{p}_2$ respectively. For price-setting purposes these firms ignore the existence of other competitors and play the mixed-strategy equilibrium of a standard captives-and-shoppers game. They enjoy expected profits equal to $v\alpha_i \prod_{j=2}^{n}(1-\alpha_j)$ for each $i \in \{1,2\}$. These are equal to the payoffs received on the equilibrium path. Examining upward list-price deviations for other firms yields Lemma 4.

Lemma 4 (Second-Stage Following a Price Deviation). *Consider a subgame in which: (i) all firms other than i maintain list prices which satisfy Lemma 3 and (ii) firm i deviates upward to a higher list price. There is a mixed-strategy Nash equilibrium of this subgame in which firms earn the same profit as they did with the original list prices.*

Lemma 4 establishes that the remaining deviations in the first stage are not profitable. For any other subgames we can pick any Nash equilibrium. Doing so we can construct a subgame perfect equilibrium that supports the play of pure strategies.

Proposition 4 (Pure Strategies on the Equilibrium Path). *If firms are strictly asymmetric, then a unique profile of prices generates an equilibrium in pure strategies. Distinct list prices are higher for larger firms. These prices are*

$$\bar{p}_1 = v \quad \textit{and} \quad \bar{p}_i = v\prod_{j=2}^{i}(1-\alpha_j) \quad \textit{for } i > 1. \tag{15}$$

Moving from larger to smaller firms, prices become relatively closer: $\bar{p}_i/\bar{p}_{i-1}$ is decreasing in i. Across the industry, profits are proportional to firms' sizes: $\pi_i = v\alpha_i\prod_{j=2}^{n}(1-\alpha_j)$.

This proposition considers the case with completely asymmetric firms. With symmetric firms (so that $\alpha_i = \alpha$ for all i and for some α) we can characterize a sequence of list prices that are unique apart from the labelling of the firms. For this symmetric case,

$$\bar{p}_i = v(1-\alpha)^{i-1} \quad \text{and} \quad \pi_i = v\alpha(1-\alpha)^{n-1}. \tag{16}$$

These outcomes are also obtained if we take the unique outcome for a fully asymmetric specification and allow the asymmetries to disappear.

Firms' (expected) profits have the feature that the awareness parameters α_i enter symmetrically for firms $i > 1$ but not for the largest firm $i = 1$. This suggests that the endogenous choice of advertising may be asymmetric (as found in Ireland, 1993). We investigate this next.

Endogenous Advertising. The n firms now participate in three stages of play. At the first stage they simultaneously choose their advertising policies: firm i chooses the (independent) proportion $\alpha_i \in [0,1]$ of buyer that are aware of its final price. The second and third stages of play follow our list-and-discount specification. Firm i's cost of advertising $C_i(\alpha_i)$ is smoothly and strictly increasing, convex, $C_i(0) = 0$, and $C_i'(0) < v$. When firms are asymmetric we order them so that $C_1'(\alpha) < \cdots < C_n'(\alpha)$ for all $\alpha \in (0,1)$.

An illustrative cost function is provided by the "random mailbox postings" of Butters (1977). Suppose that buyers are divided into $1/\Delta$ segments each of size Δ. Each segment corresponds to a mailbox. An advertisement costs $\gamma_i\Delta$ for firm i, and randomly hits one of the segments. Hence, with a total spend of $C_i(\alpha_i)$, a firm is able to distribute $C_i(\alpha_i)/(\gamma_i\Delta)$ advertisements.

It follows that $\alpha_i = 1 - (1-\Delta)^{C_i(\alpha_i)/(\gamma_i \Delta)}$. Taking the limit as $\Delta \to 0$, we observe that $(1-\Delta)^{C_i(\alpha_i)/(\gamma_i \Delta)} \to \exp(-C(\alpha_i)/\gamma_i)$. Solving for $C_i(\alpha_i)$ suggests a cost specification

$$C_i(\alpha_i) = \gamma_i \log\left(\frac{1}{1-\alpha_i}\right), \tag{17}$$

where (for asymmetric firms) we assume that $0 < \gamma_1 < \cdots < \gamma_n$.

Our solution concept is as before. We seek a profile of pure strategies (for both advertising choices and subsequent list and retail prices) along the equilibrium path, and we also look for the play of pure strategies following any first-stage deviations of advertising choices.

Definition. *A profile of advertising strategies generates an equilibrium in pure strategies if there is a subgame perfect equilibrium in which pure strategies are played both on the equilibrium path, and along the equilibrium path starting from any second-stage subgame.*

Proposition 4 characterizes firms' expected profits in any list-and-discount subgame when firms choose different advertising strategies. The same profit expressions continue to apply when some firms are the same. Given that firms are not yet ordered by their (now endogenous) choice of advertising exposure, we can write these expected profits as

$$\pi_i = \begin{cases} v\alpha_i \prod_{j\neq i}(1-\alpha_j) & \alpha_i > \max_{j\neq i}\{\alpha_j\} \text{ and} \\ v\alpha_i(1-\alpha_i)\prod_{j\notin\{i,k\}}(1-\alpha_j) & \alpha_i < \alpha_k \text{ where } \alpha_k = \max_{j\neq i}\{\alpha_j\}, \end{cases} \tag{18}$$

and where both expressions apply when firm i ties to be the largest firm.

An equilibrium (in the sense of our definition above) is generated by a pure-strategy Nash equilibrium of the simultaneous-move game in which each firm i maximizes $\pi_i - C_i(\alpha_i)$.

The sales revenue earned by a firm reacts differently to a marginal increase in its advertising reach depending on whether that firm is the largest. The largest firm sets the highest price and therefore does not worry about another firm undercutting them; it sets the monopoly price. Therefore for the largest firm, an increase in α_i increases its expected revenue linearly. In contrast, smaller firms' equilibrium prices are set to deter undercutting by larger firms. Therefore for smaller firms, there are two competing effects: fixing second-period prices, an increase in α_i scales up sales; however, it also forces its second-period price down (and that of any smaller firms because of the recursive nature of prices). In fact,

$$\frac{\partial \pi_i}{\partial \alpha_i} = \begin{cases} v\prod_{j\neq i}(1-\alpha_j) & \alpha_i > \max_{j\neq i}\{\alpha_j\} \text{ and} \\ v(1-2\alpha_i)\prod_{j\notin\{i,k\}}(1-\alpha_j) & \alpha_i < \alpha_k \text{ where } \alpha_k = \max_{j\neq i}\{\alpha_j\}. \end{cases} \tag{19}$$

For a smaller firm, revenue is decreasing in advertising exposure when a firm reaches a majority of buyers. If not, then this revenue kinks upward as α_i passes through through the maximum advertising exposure of competing firms. Specifically,

$$\frac{\lim_{\alpha_i \downarrow \max_{j\neq i}\alpha_j} \partial\pi_i/\partial\alpha_i}{\lim_{\alpha_i \uparrow \max_{j\neq i}\alpha_j} \partial\pi_i/\partial\alpha_i} = \frac{1-\max_{j\neq i}\alpha_j}{1-2\max_{j\neq i}\alpha_j} > 1, \tag{20}$$

where the inequality is strict because (once dominated strategies have been eliminated) every firm chooses positive exposure. This implies that no firm chooses its advertising reach to be exactly equal to the maximum of others, and so there is a unique largest firm.

For smaller firms, the fact advertising increases sales revenue only if $\alpha_i < \frac{1}{2}$ implies that in equilibrium all firms other than the largest restrict awareness to a minority of potential buyers. These observations are recorded in Lemma 5.

Lemma 5 (Properties of Advertising Choices). *In any pure-strategy equilibrium of the advertising game, there is a unique largest firm. Other firms advertise to a minority of buyers.*

On the revenue side, the largest firm always faces an incentive to increase its exposure. Labelling this firm as k, it is straightforward to confirm that, in equilibrium, $\partial\pi_k/\partial\alpha_k \geq 1/2^{n-1}$. Hence, if $C'(1) < 2^{n-1}$ then firm k chooses $\alpha_k = 1$ and advertises to everyone. More generally, as costs fall (e.g., allowing the cost coefficients γ_i to fall to zero in the Butters (1977) specification) the audience of the largest firm expands to give it complete market coverage.

An advertising equilibrium is characterized by the specification of a leading (and largest) firm k and n advertising choices which satisfy the $n-1$ first-order conditions

$$\frac{C_k'(\alpha_k)}{v} = \prod_{j\neq k}(1-\alpha_j) \quad \text{and} \quad \frac{C_i'(\alpha_i)}{v} = (1-2\alpha_i)\prod_{j\notin\{i,k\}}(1-\alpha_j)\ \forall i\neq k. \tag{21}$$

To fully characterize an equilibrium we also need to check for any non-local deviations. For example, one of the smaller firms $i \neq k$ has the option to deviate and choose $\alpha_i > \alpha_k$ and become the largest firm. The proof of Proposition 5 checks such remaining details.

Proposition 5 (Pure-Strategies on Path: Endogenous Advertising). *There is at least one equilibrium of the advertising-then-pricing game in which firms choose pure strategies along the equilibrium path, and also do so following any first-stage deviation in advertising choice.*

In any such equilibrium, one firm chooses a strictly higher advertising level than all the others, sets a list price equal to the maximum willingness to pay of consumers, and only sells to buyers who are uniquely aware of its product. Other firms advertise to at most half of potential buyers.

In equilibrium a leading firm advertises distinctly more than others. Proposition 5 does not identify this firm. If the advertising cost functions are not too different then any firm can play this role. If they are different then the leading firm enjoys relatively lower advertising costs.[19] The other minority-audience firms can, however, be ordered given the structure of the advertising cost functions. For example, if $k = 1$ then advertising choices satisfy $\alpha_1 > \cdots > \alpha_n$.

If firms are symmetric ($C_i(\alpha_i) = C(\alpha_i)$ for all i) then the first-order conditions simplify appreciably. Writing α for the common advertising choice of the smaller firms,

$$\frac{C'(\alpha_k)}{v} = (1-\alpha)^{n-1} \quad \text{and} \quad \frac{C'(\alpha)}{v} = (1-2\alpha)(1-\alpha)^{n-2}. \tag{22}$$

[19]Formally: there is some $k^\star$ such that there is an equilibrium in which any $k \in \{1, \ldots, k^\star\}$ leads the industry.

A special case is when advertising is free. There is, of course, a pathological equilibrium in which multiple firms choose $\alpha_i = 1$ and subsequent prices are competed down to marginal cost. Putting this aside (or by allowing costs to be close to free) the "free advertising" case yields $\alpha = \frac{1}{2}$ for $n-1$ firms, and complete coverage for one firm.

Proposition 6 (Equilibrium with Free Advertising). *If advertising is free then, in an equilibrium in which firms earn positive profits, the largest firm chooses maximum advertising exposure, while others advertise to half of potential buyers. Labelling firms appropriately, final retail prices satisfy $\bar{p}_i = v/2^{i-1}$. The largest firm earns twice the profit of each smaller firm.*

As a final case, we consider the cost specification derived from random mailbox postings suggested by Butters (1977). For the cost function of (17), the marginal cost of increased advertising satisfies $C'(\alpha) = \gamma/(1-\alpha)$. Setting $\gamma = 1$ without loss of generality (for this cost coefficient only matters relative to the valuation v of buyers for the product) and requiring $v > 1$ (otherwise all firms choose zero advertising) the relevant first-order conditions become

$$\frac{1}{v(1-\alpha_k)} = (1-\alpha)^{n-1} \quad \text{and} \quad \frac{1}{v} = (1-2\alpha)(1-\alpha)^{n-1}. \tag{23}$$

These equations solve recursively. Substituting the second into the first, we find that $\alpha_k = 2\alpha$: no matter what the level of cost, the large firm has twice the advertising reach of all smaller firms. The solution for α satisfies the natural comparative-static property that α is increasing in the product valuation v, and so is decreasing in the advertising cost parameter γ.[20]

Proposition 7 (Equilibrium with Random Mailbox Postings). *If the cost of advertising reach is determined by a random mailbox postings technology, so that $C(\alpha) = -\gamma \log(1-\alpha)$, and firms are symmetric, then the largest firm chooses advertising awareness equal to double that of the competing small firms. Advertising is increasing in buyers' willingness to pay.*

Related Models. The "independent awareness" advertising technology that we use here is, of course, not new to this paper. An under-appreciated extension of the model of sales was developed by Ireland (1993) in which first-stage (costless) advertising choices are followed by a single stage of pricing. His pricing stage (very naturally) generates an equilibrium in mixed strategies with exactly the same payoffs as those reported here. This naturally implies that the first-stage equilibrium advertising choices are asymmetric, as described in Proposition 6. Other authors have studied versions of the model of sales but with a pre-pricing stage in which firms determine their captive shares and have also found asymmetric equilibrium advertising outlays (Chioveanu, 2008; Ronayne and Taylor, 2019). In contrast to these papers, our result maintains the prediction of asymmetric advertising intensities while allowing for the on-path play of pure strategies. Moreover, we show that the two-to-one advertising ratio (in which the lead firm advertises to twice the audience of other firms) does not require free advertising; it emerges via the random-mailbox-postings advertising technology.

[20]An explicit solution is easily obtained when $n = 2$: $\alpha = \frac{3}{4} - \frac{1}{4}\sqrt{1 + \frac{8}{v}}$.

5. A Model of Buyer Search

For all three models considered so far, the access of buyers to prices has been either exogenous or influenced by firms' marketing choices. Here we allow for endogenous consumer search.

We pair our two-stage pricing approach with the model of Janssen and Moraga-González (2004), in which buyers pay for costly price quotations. Our predictions differ from those with single-stage pricing. Under reasonable parameters, buyers obtain either one or two price quotations and so are, endogenously, either captive to a single firm or are pairwise shoppers. Using pure strategies, firms post a distinct sequence of prices. Under the equivalent model with single-stage pricing, there can also be equilibria in which buyers obtain no more than one quotation. We show that, under two-stage pricing, such equilibria exist only for narrow parameter values, and involve no search at all if there are no exogenous shoppers.

Our comparative-static predictions also differ from those that hold under single-stage pricing. Fixing the search strategies of buyers, we find that the average price posted by the firms (this is equal to the average price paid by captive buyers) does not rise with entry to the industry (cf. Janssen and Moraga-González, 2004). We also find that the average price paid paid across all buyers is independent of the number of competitors, and is equal to the median price posted. Allowing for endogenous search, increases in either the number of firms or the search cost raise the fraction of pairwise shoppers, lower the prices posted, and lower the average price paid.

A Pricing Game with Costly Fixed-Sample Search. On the supply side, we follow the specification of earlier sections. There are n symmetric firms with constant marginal cost normalized to zero. They post list prices, and then set (weakly lower) final retail prices.

On the demand side, a fraction $\lambda_S \in [0, 1]$ of buyers are shoppers who see all n prices. Following Janssen and Moraga-González (2004) and Burdett and Judd (1983), others are "searchers" who use a fixed-sample technology to obtain price quotations.[21] Moving simultaneously, each searcher pays κq to obtain $q \in \{0, 1, \ldots, n\}$ quotations, where $0 < \kappa < v$. This randomly reveals (without replacement) q out of the n price offers. The buyer selects the cheapest offer.

We imagine a situation in which buyers search at the same time as firms engage in pricing. Such a game has no proper subgames. We seek a solution, however, which would be supported by a subgame perfect equilibrium of the pricing game if the search strategies of buyers were known. To do this formally, we specify a game in which buyers move first and simultaneously choose search policies. These policies are observed by firms, before they begin their own stages of play. The first pricing stage is now the start of a proper subgame. Any individual buyer has no measurable influence on future play in the search-choice stage and so acts as though moving simultaneously. This enables us to use subgame perfection as a solution concept.

[21]We can allow for $\lambda_S = 0$ so that there are no exogenous shoppers, and so all buyers are searchers. In the original studies of costly search, the assumption that $\lambda_S > 0$ was justified by appealing to the fact that some people enjoy shopping (Stahl, 1989) or have no opportunity cost of time (Janssen and Moraga-González, 2004).

Our notation follows Janssen and Moraga-González (2004): μ_q is the proportion of searchers who pay for q quotations. Using a single-stage specification for firms' pricing, they identified three possibilities for equilibria: (i) a low search intensity equilibrium in which $\mu_0, \mu_1 > 0$ and $\mu_0 + \mu_1 = 1$; (ii) a moderate search intensity equilibrium in which $\mu_1 = 1$; and (iii) a high search intensity equilibrium in which $\mu_1, \mu_2 > 0$ and $\mu_1 + \mu_2 = 1$.[22]

Expanding notation further, for this third case the fractions of potential buyers who see only a price quotation from firm i (and so are captive) or compare the prices of firms i and j are

$$\lambda_i = \frac{\mu_1(1-\lambda_S)}{n} \quad \text{and} \quad \lambda_{ij} = \frac{2\mu_2(1-\lambda_S)}{n(n-1)}. \tag{24}$$

We seek subgame perfect equilibria in which prices are chosen as pure strategies. Specifically, an "equilibrium" corresponds to a search strategy (which specifies that a fraction μ_q of buyers request q price quotations) and a set of prices such that it is optimal for a buyer to request q quotations when $\mu_q > 0$, and the set of prices meet our earlier definition of equilibrium.

Low and Moderate Intensity: Searching Once or Never. A familiar property of costly search is that buyers obtain at most two quotations, and some obtain only one. The rough logic is that there are (at least weakly) decreasing returns to increasing the sample size of a search, and so there is either a single optimal sample size q (so that $\mu_q = 1$ for this q) or neighboring sample sizes q and $q+1$ are both optimal (so that $\mu_q + \mu_{q+1} = 1$).[23] If all buyers obtain at least two price quotations ($q \geq 2$) then every firm is sure to face head-to-head competition with at least one other firm. This forces prices (also in our two-stage game) to zero. However, if prices are zero then consumers optimally obtain only a single quotation; a contradiction.

Lemma 6 (Number of Quotations in Equilibrium). *In equilibrium $\mu_{q>2} = 0$ and $\mu_1 > 0$: searchers gather at most two quotations, and some gather exactly one.*

Here we consider buyers who obtain one quotation or none, so that $\mu_2 = 0$. This combines the cases of low search intensity ($0 < \mu_1 < 1$: some buyers never search) and moderate search intensity ($\mu_1 = 1$: everyone obtains one quotation) from Janssen and Moraga-González (2004). This is a world of captives and shoppers, where the mass of buyers loyal to i is $\lambda_i = \mu_1(1-\lambda_S)/n$. We know (Proposition 1) that $n-1$ firms fully exploit captives while the nth firm chooses a list price $p^\dagger$ low enough to dissuade discounts from others. From eq. (1),

$$p^\dagger = \frac{\lambda_i v}{\lambda_i + \lambda_S} = \frac{\mu_1(1-\lambda_S)v}{\mu_1(1-\lambda_S)+\lambda_S n} \quad \Leftrightarrow \quad \frac{v-p^\dagger}{n} = \frac{\lambda_S v}{\mu_1(1-\lambda_S)+\lambda_S n}. \tag{25}$$

Turning to optimal search, a quotation costs κ and earns surplus $v - p^\dagger$ if the "limit price" firm n supplies the quotation, which happens with probability $1/n$; this generates the expression on the right. The fact that a buyer is searching for a single lowest price means that the gains from search (just like the costs) are linearly increasing in the number of quotations q; the probability that q searches without replacement find the cheapest firm is q/n. This means that if it is

[22] The low-intensity equilibrium, $\mu_0 > 0$, would not exist if the first search were free, as in Stahl (1989).
[23] When $n = 2$ we can also construct situations in which buyers are indifferent between three options $q \in \{0, 1, 2\}$.

strictly preferred to search for one quotation rather than none, then it is strictly optimal to search for quotations from all n suppliers. To construct an equilibrium in which $\mu_1 > 0$ but $\mu_2 = 0$ requires a searcher to be exactly indifferent between searching and not. Solving for μ_1 is straightforward; but we need to check that $\mu_1 \in (0, 1]$.

Proposition 8 (Searching Once or Never). *If $n < v/\kappa < n - 1 + (1/\lambda_S)$ then there is an equilibrium in which buyers search for a single quotation with probability*

$$\mu_1 = \frac{\lambda_S}{1 - \lambda_S} \left(\frac{v}{\kappa} - n \right) \tag{26}$$

but otherwise do not search. Firms play pure strategies on the equilibrium path. One firm sets a low list price $p^\dagger = v - n\kappa$, while all others charge v. There are no second-stage discounts.

Janssen and Moraga-González (2004) defined a "moderate search intensity" equilibrium as one in which every buyer obtains exactly one quotation, so that $\mu_1 = 1$. For generic parameters (the exception is when $n - 1 + (1/\lambda_S) = v/\kappa$) there is no such equilibrium.[24] Therefore, our solution corresponds to their "low search intensity" equilibrium. Interestingly, they (in their Propositions 4–5) established the existence of such an equilibrium when n is large. In contrast, we also require n to be sufficiently small.[25] With single price quotations, the equilibrium property (with pure-strategy play) that only one firm chooses a price lower than v means that buyers are searching for the proverbial needle in the haystack when n is large. Hence, even for low cost-to-value ratios, buyers can find it not worthwhile to search at all.

More generally the fraction of those who search is decreasing in n. An increase in n also pushes down the lowest price while making the highest prices (equal to v) more frequent, and so makes the distribution of prices riskier while the average price remains constant.[26]

Our final observations concern the fraction of shoppers who exogenously see all prices. Suppose $n < v/\kappa$, so that a buyer would search if at least one firm gives the product away. If λ_S is small, then the inequality $v/\kappa < n - 1 + (1/\lambda_S)$ required for existence holds. However, as this fraction is negligible then trade collapses: $\mu_1 \downarrow 0$ as $\lambda_S \downarrow 0$ i.e., endogenous search falls to zero as the exogenous searchers are removed from the pool of potential buyers. In essence this says that a low search intensity equilibrium is a no search equilibrium.

High Intensity: Searching Once or Twice. We have shown that lower (at most one quotation) search intensities either are not chosen in equilibrium or involve negligible search. The

[24] For our list-prices-and-discounts game (with pure-strategy play) we rule out Propositions 1–3 of Janssen and Moraga-González (2004, Section 3, pp. 1094–1098) which rely on their moderate-intensity equilibrium. In their world, the use of single-stage pricing à la Varian (1980) means that firms choose mixed strategies, and so there are many different possible prices. This implies that there are decreasing returns to increased consumer search, which contrasts with the linearity that we obtain here. One way to recover existence for a non-degenerate set of parameters would be to assume that search costs are strictly increasing, rather than constant.

[25] Whereas n needs to be sufficiently small (to satisfy the first inequality in Proposition 8) it cannot be too small (because of the second inequality): if $v/\kappa > (1 + \lambda_S)/\lambda_S$ then that second inequality fails for n small enough. This is because too few firms can make the probability that a searcher finds the low price to be so high that all searchers wish to search. The fact that the low price rises as n falls tempers this effect, but does not undo it.

[26] These results reinforce those reported in Proposition 5 of Janssen and Moraga-González (2004).

more interesting case is when some searchers ask for two price quotations so that each firm faces a positive probability of going head-to-head with any other firm. This (as it does in the advertising model of Section 4) ensures that any equilibrium with the on-path play of pure strategies must involve n distinct prices. We order the firms such that $\bar{p}_1 > \cdots > \bar{p}_n$.

Fixing buyers' search strategies, we now characterize the staircase of prices that satisfy "no undercutting" constraints: firm $i < n$ with list price $\bar{p}_i$ does not wish to undercut (at the discounting stage) the list price $\bar{p}_{i+1}$ and so capture those additional searchers who see quotations from both firms i and $i+1$. For $i < n-1$, this constraint is

$$\left(\lambda_i + \sum\nolimits_{j<i}\lambda_{ij}\right)\bar{p}_i \geq \left(\lambda_i + \lambda_{i(i+1)} + \sum\nolimits_{j<i}\lambda_{ij}\right)\bar{p}_{i+1}. \tag{27}$$

On the left-hand side, maintaining list price $\bar{p}_i$ wins sales from the mass λ_i of customers who see only this price, and also wins sales $\sum_{j<i}\lambda_{ij}$ from comparisons with higher-priced rivals. On the right hand side, undercutting $\bar{p}_{i+1}$ grabs extra sales of $\lambda_{i(i+1)}$ (the searchers who compare i's price against the next cheapest firm). If this constraint held as a strict inequality then firm $i+1$ could safely raise $\bar{p}_{i+1}$, and so (27) holds with equality. For firm $n-1$, there is an additional incentive to undercut firm n: by charging below $\bar{p}_n$ this firm can also capture the shoppers (who see all prices).[27] The presence of shoppers only influences the lowest price $\bar{p}_n$ in the sequence. We simplify exposition (with no real loss of insight) by setting $\lambda_S = 0$. Finally, the highest list price is $\bar{p}_1 = v$ as this firm sells only to captives. These equations pin down a unique profile of prices that are part of an equilibrium with the on-path play of pure strategies. The terms λ_i and λ_{ij} are, in turn, determined endogenously by the search strategies of buyers.

Just as before, there is no incentive (by construction) for any firm to undercut at either stage. Checking for any profitable deviation upwards at the list-price stage (by constructing equilibria in the appropriate subgames) yields Proposition 9.

Proposition 9 (Equilibrium Prices with Search for Two Quotations). *Set $\lambda_S = 0$. Suppose that the search strategy satisfies $\mu_1 > 0$, $\mu_2 > 0$ and $\mu_1 + \mu_2 = 1$. For the two-stage pricing game, there is a unique profile of list prices*

$$\bar{p}_i = \frac{\mu_1(n-1)v}{\mu_1(n-1) + 2(1-\mu_1)(i-1)} \tag{28}$$

played as pure strategies as part of a subgame perfect equilibrium. There are no discounts. The (distinct) prices are decreasing in the proportion $\mu_2 = 1 - \mu_1$ of buyers who search for a second price quotation. If n is odd, then the median price is equal to $\mu_1 v$.

The average price charged is lower in an industry with $n > 2$ competitors than in a duopoly.

Each firm earns profit of $\mu_1 v/n$, and so the average paid by a buyer is equal to $\mu_1 v$.

The final claims concern profitability and so (equivalently) the average price paid. A fraction of μ_1/n of customers are effectively captive to each firm. A fraction $1-\mu_1$ are pairwise shoppers.

[27] For firm $n-1$ and $\lambda_S > 0$: $(\lambda_{n-1} + \sum_{j<n-1}\lambda_{j(n-1)})\bar{p}_{n-1} = (\lambda_{n-1} + \lambda_{n-1(n)} + \sum_{j<n-1}\lambda_{j(n-1)} + \lambda_S)\bar{p}_n$.

The most expensive firm loses any comparisons, and so earns the profit from exploitation of captive customers. The price construction means that all firms earn equal expected profit. The average price paid does not depend directly on the number of competitors. Instead, it depends on the strategy of buyers: more intensive search lowers the distribution of prices posted.

Although the average price paid is independent of n, the distribution of prices is not. Notably (cf. Janssen and Moraga-González, 2004) the average price posted is not increasing in n. Beyond the statement made in the proposition, we can also show that the average is decreasing for $n \in \{2, 3, 4\}$ before calculations become cumbersome.[28]

Now we consider endogenous search. From Proposition 9 we can work out the expected gain to acquiring one or two price quotations. An equilibrium is obtained when the gain from the second quotation (this is weakly lower than from the first quotation) equals κ.

Consider, for example, the the duopoly case ($n = 2$) for which the two prices are $\bar{p}_1 = v$ and $\bar{p}_2 = \mu_1 v/(2 - \mu_1)$. A single quotation finds the cheaper with firm with probability 1/2 and generates consumer surplus $v - \bar{p}_2$. Two quotations find this consumer surplus with certainty. For this special case of $n = 2$, the marginal benefit of both the first and second quotations is equal to $(v - \bar{p}_2)/2$.[29] Equating this to the quotation cost κ yields an equilibrium. The case where $n = 3$ is also relatively tractable. We report both the duopoly and triopoly cases here.

Proposition 10 (Equilibrium Search in a Duopoly and Triopoly). *Set $\lambda_S = 0$. Consider an equilibrium in which potential buyers acquire either one or two price quotations.*

For a duopoly with $v > 2\kappa$, there is a unique equilibrium which satisfies

$$\mu_2 = 1 - \mu_1 = \frac{\kappa}{v - \kappa}, \quad \frac{\bar{p}_1 + \bar{p}_2}{2} = v - \kappa, \quad \text{and average price paid} = \mu_1 v = \frac{v(v - 2\kappa)}{v - \kappa}. \tag{29}$$

Buyers are indifferent between searching once, twice, or never; they earn zero expected payoff.

For a triopoly with $v > 3\kappa/2$, there is a unique equilibrium which satisfies

$$\mu_2 = \frac{\kappa}{(2/3)v - \kappa}, \quad \frac{\bar{p}_1 + \bar{p}_2 + \bar{p}_3}{3} = v - \kappa - \frac{\kappa v}{2v - 3\kappa}, \quad \text{and av. price paid} = \frac{v(2v - 6\kappa)}{2v - 3\kappa}. \tag{30}$$

A move from duopoly to triopoly or an increase in the search cost κ raises the intensity of search and lowers the average price charged and the average price paid.

[28] Proposition 6 of Janssen and Moraga-González (2004), which says that the average price charged increases with n, does not hold. Under the symmetric mixed strategies of the single-stage game it is uncertain whether a particular firm will win shoppers. As the number of firms grows, the probability of being the cheapest falls exponentially. Firms react to this exponential fall in equilibrium by shifting mass to higher prices. This force is not present under our equilibrium price profile: the lowest price $\bar{p}_n$ internalises this force completely.

[29] We can construct an equilibrium in which some buyers do not search at all. Formally, for any equilibrium in which $\mu_1 + \mu_2 = 1$ and any $\tilde{\mu}_0 > 0$ there is an equilibrium in which all three search sizes $q \in \{0, 1, 2\}$ are used, satisfying $\tilde{\mu}_q = \mu_q(1 - \tilde{\mu}_0)$ for $q \in \{1, 2\}$. A slight change to our model eliminates this multiplicity. If we assume that the second quotation is slightly more expensive than the first (so that the cost of search exhibits decreasing returns) then once again any equilibrium can only involve two adjacent search strategies.

Proposition 10 reports that intensity increases as search becomes more costly. This follows from the equilibrium requirement that searchers must be indifferent between one and two quotations: greater intensity pushes prices down, compensating searchers for their increased costs.[30]

Many Suppliers. Given the search strategy of buyers, we have equilibrium prices for any n. However, various expressions become cumbersome for larger oligopolies. To make progress, we consider the limiting case where there are large number of suppliers. To do so, we specify a mass of firms, and a distribution $F(\cdot)$ of prices across those firms. Given a search strategy across one or two quotations from potential buyers, the distribution satisfying Lemma 7 ensures that firms are indifferent between prices across the relevant support.

Lemma 7 (Equilibrium Price Distribution with Many Suppliers). *Fix the search strategy of buyers, and consider the unique profile of prices that generates an equilibrium in pure strategies. Allowing the number of firms to grow large, the distribution of prices converges to a continuous distribution with support on* $[\mu_1 v/(2-\mu_1), v]$. *Writing* $F(\cdot)$ *for this distribution, and* $F_{\min}(\cdot)$ *for the distribution of the minimum* $\min\{p', p''\}$ *of two randomly drawn prices,*

$$F(p) = 1 - \frac{\mu_1}{1-\mu_1}\frac{v-p}{2p} \quad \text{and} \quad F_{\min}(p) = 1 - \left(\frac{\mu_1}{1-\mu_1}\frac{v-p}{2p}\right)^2. \tag{31}$$

The expected payment from (i) a single quotation and (ii) two quotations satisfy

$$\mathrm{E}[p] = \frac{v}{2}\frac{\mu_1}{1-\mu_1}\log\left(\frac{2-\mu_1}{\mu_1}\right) \tag{32}$$

$$\mathrm{E}[\min\{p', p''\}] = \frac{v}{2}\left(\frac{\mu_1}{1-\mu_1}\right)^2\left(\frac{2(1-\mu_1)}{\mu_1} - \log\left(\frac{2-\mu_1}{\mu_1}\right)\right). \tag{33}$$

Here we highlight an important connection between the equilibria of our model and those in the literature. The mixed equilibria of single-stage models are not ex post Nash for finite n. But as $n \to \infty$, the equilibria of a large class of games become ex post Nash (Kalai, 2004, Theorem 1). The mixed-strategy equilibria of the single-stage pricing models we have examined are no exception. In fact, as $n \to \infty$ in our model, prices organize themselves in such way as to asymptotically produce the exact same distribution, F of (31), as reported in those studies.[31] Hence both paradigms are robust to ex post deviations in the limit. The difference is that our model's equilibrium is ex post Nash for all n, not only when $n \to \infty$.

Given the equivalence of our equilibrium with that in the literature for a large number of firms, we derive optimal search and give our final result by following previous papers closely, e.g.,

[30] The need for this indifference condition is artefact of there being one level of search cost for everyone. Suppose instead there were two searcher types: low-cost and high-cost with respective marginal costs of search κ_H and $\kappa_L < \kappa_H$. It is then straightforward to construct an equilibrium for a non-degenerate set of parameters in which: (i) prices are played with pure strategies; (ii) low-cost (high-cost) searchers strictly prefer to gather two (one) quotation i.e., there are no indifference conditions; and (iii) prices and search intensities are independent of search costs. Whether such an equilibrium exists of course depends on κ_L and κ_H.

[31] In Janssen and Moraga-González (2004), the mixed strategy they derive for their high search intensity's equilibrium (see their equation 8) is ex post Nash only when $n \to \infty$. The same expression (without exogenous shoppers) can also be found in Burdett and Judd (1983, equation 2) where n is large throughout.

Burdett and Judd (1983, Section 3.2) and (Janssen and Moraga-González, 2004, pp. 1104–1109). The cost of an additional quotation must be equal to the expected reduction in price:

$$\kappa = \mathrm{E}[p] - \mathrm{E}[\min\{p', p''\}] = \frac{v}{2}\frac{\mu_1}{1-\mu_1}\left(\frac{1}{1-\mu_1}\log\left(\frac{2-\mu_1}{\mu_1}\right) - 2\right). \tag{34}$$

The right-hand side is single-peaked, rising from zero at $\mu_1 = 0$ and falling back to zero at $\mu_1 = 1$. This implies that if κ is large there is no solution; but if κ is sufficiently small there are two equilibrium values for μ_1, as detailed in Proposition 11.

Proposition 11 (Equilibrium Search with Many Suppliers). *Consider an equilibrium with a continuum of suppliers and endogenous search. There is some $\bar{\kappa}$ such that if $\kappa < \bar{\kappa}$ then there are two equilibria. Both involve greater buyer search for two quotations (and so lower industry profit, lower average price charged, and lower average price paid) than for the duopoly case.*

As discussed, our predictions depart markedly from the literature for finite n, and coincide when $n \to \infty$. Proposition 11 establishes that the incentive to acquire a second quotation lies everywhere below the incentive when $n = 2$. This means that equilibria in the two cases are distinct, involving a higher search intensity (and so lower prices) when the number of competitors is very large rather than very small. This prediction is distinct from the single-stage model of Janssen and Moraga-González who predict that the expected price charged is the same with a duopoly and with many firms (see their Proposition 8).

6. Concluding remarks

Our two-stage model predicts a unique profile of distinct prices played with pure strategies. We therefore generate stable price dispersion, in line with the empirical evidence. The model is highly portable and conducive to deeper analyses as demonstrated by our applications, where we generate new insights in models of sales, prominence, advertising and search. The traditional single-stage framework has been applied to many other research areas too, including: price discrimination (Armstrong and Vickers, 2018a; Fabra and Reguant, 2018); product substitutability (Inderst, 2002); sequential consumer search (Stahl, 1989); switching costs (for a review, see Farrell and Klemperer, 2007); strategic clearing-houses such as comparison websites (Baye and Morgan, 2001, 2009; Moraga-González and Wildenbeest, 2012; Ronayne, 2019; Shelegia and Wilson, 2017); competition with boundedly-rational consumers (Carlin, 2009; Chioveanu and Zhou, 2013; Heidhues, Johnen, and Kőszegi, 2018; Inderst and Obradovits, 2018; Piccione and Spiegler, 2012). As such, our novel pricing game promises to facilitate and enrich a broad portfolio of inquiry.

Omitted Proofs

Proof of Lemma 1. This follows from the argument in the main text. □

Proof of Lemma 2. The conditions of Theorem 5 of Dasgupta and Maskin (1986, p. 14) are met, and guarantee the existence of a mixed-strategy Nash equilibrium.[32]

Suppose that $p_i \sim F_i(\cdot)$, and write $\bar{s}_i$ for the upper bound to the support of this mixed strategy.

Claim (i). No firm places an atom strictly below its list price. This absence of atoms implies that the expected profit of a firm is continuous in its price except at others' list prices.

An atom at $p_i < \bar{p}_i$ is justified only if it can capture the shoppers. No other firm $j \neq i$ prices just above p_i; it would be better for j to capture the atom. he only firms that might set $p_j = p_i$ are strictly more efficient ($c_j < c_i$) because they (by our tie-break rule) can win at tied prices; but that means that firm i loses against them anyway. We conclude that firm i can raise p_i locally without losing sales, and yet increasing the profit from captives; a contradiction.

Claim (ii). Firm n performs strictly better than it does by exploiting only captives.

For $i < n$ prices strictly below $p^\dagger_{n-1}$ are strictly dominated. No firm places an atom at $p^\dagger_{n-1}$. This means that firm n can capture all shoppers by setting $p_n = p^\dagger_{n-1}$, and so n's expected profit weakly exceeds $(p^\dagger_{n-1} - c_n)(\lambda_0 + \lambda_n) > (p^\dagger_n - c_n)(\lambda_0 + \lambda_n) = (v - c_n)\lambda_n \geq (\bar{p}_n - c_n)\lambda_n$.

Claim (iii). A firm's pricing mixture includes its list price: $\bar{s}_i = \bar{p}_i$ for all i.

If $\bar{p}_j < \bar{s}_i < \bar{p}_i$ then firm i would never choose $p_i \in (\bar{p}_j, \bar{p}_i)$: sales are only to captives (firm j is guaranteed to be cheaper) and firm i would do better to raise price further. Suppose instead that $\bar{p}_j > \bar{s}_i$ for all j. For the same reason, firm j never chooses $p_j \in (\bar{s}_i, \bar{p}_j)$. From claim (i) there is no atom at $\bar{s}_i$. Firm i could again raise price from $\bar{s}_i$ without losing sales. We conclude that $\bar{s}_i < \bar{p}_i$ can be optimal only if some j places an atom at $\bar{p}_j = \bar{s}_i$. This must be firm n (all others satisfy $\bar{p}_j = v > \bar{s}_i$). Firm i does not place an atom at $\bar{s}_i$, from claim (i). Hence firm n sells only to captives with an atom at $\bar{p}_n$. This contradicts claim (ii).

Claim (iv). Firm $i < n$ earns an expected payoff of $\lambda_i(v - c_i)$.

Suppose that $\bar{p}_n < v$. From claim (iii), the mixed strategy of $i < n$ includes its maximum price at which no shoppers buy and so firm i earns $\lambda_i(v - c_i)$ from sales to captive customers.

Suppose instead that $\bar{p}_n = v$. If every firm were to place an atom at v then at least one firm would undercut the others. Hence some firm j places no atom at v. Other firms $i \neq j$ are willing to price at or close to v, and are always beaten on price by firm j. These firms earn

[32] A condition is that the sum of payoffs (here, aggregate profit) is upper semi-continuous in actions (here, prices). Industry revenue is continuous. Consider costs when the allocation of output changes discontinuously as prices change. For upper semi-continuity we require the allocation to maximize industry profit, and so minimize aggregate cost, at any tied prices. This is achieved by breaking ties in favor of efficient suppliers.

$\lambda_i(v - c_i)$. If $n \neq j$, then firm n would earn $\lambda_n(v - c_n)$, which contradicts claim (ii). Hence $j = n$, and all firms $i < n$ earn $\lambda_i(v - c_i)$ as claimed. Note that all firms $i < n$ must place an atom at v. If some $i < n$ were to join n by omitting the atom, then once again firm n's expected profit would fall to $\lambda_n(v - c_n)$, in contradiction to claim (ii).

Firm i's profit can exceed $\lambda_i v$ only if all other more efficient firms have an atom at v.

Claim (v). Firm n earns an expected payoff of $(\lambda_0 + \lambda_n)(p^\dagger_{n-1} - c_n)$.

Suppose that firm n never prices below some $p > p^\dagger_{n-1}$. Firm $n-1$ could price within the interval $(p^\dagger_{n-1}, \min\{p, p^\dagger_{n-2}\})$ and sell to all shoppers, at a price yielding a profit that strictly exceeds $(\lambda_S + \lambda_{n-1})p^\dagger_{n-1} = \lambda_{n-1}v$. This contradicts claim (iv). Hence the mixed strategy of firm n extends down to $p^\dagger_{n-1}$, which captures all shoppers and yields the claimed profit. □

Proof of Proposition 1 and Corollaries. By construction, no high-priced firm has an incentive to undercut firm n in the second stage, and so no incentive to post a lower price. The only other first-stage deviation is an increase in the price of firm n. This yields no change in payoff, given the play in that subgame described in Lemma 4.

The three corollaries all follow from the accompanying discussions in the main text. □

Proof of Proposition 2. Consider a strategy profile in which the three firms post the list prices as stated, and then (on the equilibrium path) choose $p_i = \bar{p}_i$ in the second stage.

In the second stage, no firm is able to raise price. Non-prominent firms serve all available customers, and so face no incentive to discount. Finally, the stated list prices ensure that the prominent firm has no incentive to undercut any competitors. In the first stage, the same arguments ensure that nothing can be gained from advertising a lower first-stage list price.

It remains to check for an upward deviation in the list prices of non-prominent firms.

Recalling that $\bar{p}_1 > \bar{p}_3 > \bar{p}_2$, consider a deviation by firm 3 to a higher list price $\tilde{p}_3 > \bar{p}_3$. The following strategies constitute a Nash equilibrium in such a subgame. Firm 2 charges $p_2 = \bar{p}_2$. Firms 1 and 3 mix continuously over the interval from $\bar{p}_3$ to $\tilde{p}_3$ using the distributions

$$F_1(p) = \frac{p - \bar{p}_3}{p} \quad \text{and} \quad F_3(p) = \frac{(\phi_1 + \phi_3)p - \phi_1 v}{\phi_3 p}, \tag{35}$$

placing any residual mass at $\bar{p}_1$ and $\tilde{p}_3$, respectively. It is straightforward to confirm that firms 1 and 3 earn profits $\phi_1 v$ and $\phi_3 \bar{p}_3$ respectively; there is no gain relative to the equilibrium path.

Suppose that firm 2 deviates upward to $\tilde{p}_2 > \bar{p}_2$ but where $\tilde{p}_2 \leq \bar{p}_3$. These strategies constitute a Nash equilibrium. Firm 3 charges $p_3 = \bar{p}_3$. Firms 1 and 2 mix from $\bar{p}_2$ to $\tilde{p}_2$ via

$$F_1(p) = \frac{p - \bar{p}_2}{p} \quad \text{and} \quad F_2(p) = \frac{(\phi_1 + \phi_2 + \phi_3)p - \phi_1 v}{\phi_2 p}, \tag{36}$$

placing residual mass at $\bar{p}_1$ and $\tilde{p}_2$, respectively. Firms 1 and 2 earn ϕv and $\phi_2 \bar{p}_2$.

Finally, if $\tilde{p}_2 > \bar{p}_3$ then the following strategy profile constitutes a Nash equilibrium. Firm 1 mixes just as before. Firm 3 mixes from $\bar{p}_2$ up to $\pi_1 v/(\phi_1 + \phi_2)$ using the distribution

$$F_3(p) = \frac{(\phi_1 + \phi_2 + \phi_3)p - \phi_1 v}{\phi_3 p}. \tag{37}$$

Firm 3 mixes from $\pi_1 v/(\phi_1 + \phi_2)$ up to $\tilde{p}_2$ using the distribution

$$F_2(p) = \frac{(\phi_1 + \phi_2)p - \phi_1 v}{\phi_2 p}, \tag{38}$$

while placing residual mass at $\tilde{p}_2$. Firm 2 earns $\phi_2 \bar{p}_2$ just as on the equilibrium path. Hence, following each possible deviation we have specified a Nash equilibrium of the subgame in which there is no gain for the player deviating at the first stage.

This shows that the prices in the proposition are supported in a subgame perfect equilibrium.

In the text, we reported a second profile of list prices (in which the larger non-prominent firm posts an intermediate price) which satisfied the necessary conditions discussed in the text:

$$\bar{p}_1 = v, \quad \bar{p}_2 = \frac{\phi_1 v}{\phi_1 + \phi_2}, \quad \text{and} \quad \bar{p}_3 = \frac{\phi_1 v}{\phi_1 + \phi_2 + \phi_3}, \tag{39}$$

We now show that these prices cannot be played as pure strategies on the equilibrium path of a subgame perfect equilibrium. If they were, then on that path each firm i would set $p_i = \bar{p}_i$ (by construction, the prominent firm has no incentive to undercut) and firm 3 would earn a payoff $\phi_3 \phi_1 v/(\phi_1 + \phi_2 + \phi_3)$. We will show that firm 3 has a profitable deviation in the first stage.

Specifically, consider the subgame following a deviation by firm 3 to a list price $\tilde{p}_3$ where

$$\frac{\phi_1 v}{\phi_1 + \phi_3} > \tilde{p}_3 > \frac{\phi_1 v}{\phi_1 + \phi_2} = \bar{p}_2 > \bar{p}_3, \tag{40}$$

No firm places an atom strictly below its list price: if a firm did then no competitor would never price at or just above this atom, and so the firm could safely move the atom upward.

There can be no equilibrium in pure strategies: this would involves prices equal to list prices, and the prominent firm would find it profitable to undercut $\bar{p}_2$ and so capture all of the business.

The prominent firm uses a mixed strategy: if it were were to play the pure strategy $p_1 = \bar{p}_1 = v$ then the opponents would play their list prices. That mixed strategy can place an atom at $\bar{p}_1 = v$, and otherwise mixes continuously over the support of the others' mixed strategies.

For the prominent firm, prices strictly below $\bar{p}_3$ are strictly dominated, as are prices above $\bar{p}_2$ but below $\phi_1 v/(\phi_1 + \phi_3)$. Either firm $i \in \{2, 3\}$ can secure all customers by charging $p_i = \bar{p}_3$ and so can guarantee a payoff of $\phi_i \bar{p}_3 > 0$. Take the highest price charged by any non-prominent firm. This wins customers with strictly positive probability (as it must to generate a positive expected profit) only if the prominent firm prices above it with strictly positive probability. Thus the prominent firm places an atom at $\bar{p}_1$. This implies that payoff of this firm equals $\phi_1 v$.

Excluding the atom at $\bar{p}_1 = v$, consider the support of the prominent firm's (continuous) mixed strategy. This lies within the joint support of the competitors' strategies: any other price can be safely raised (that is, without losing sales) which strictly raises profit. The support of any competitor lies within the support of the prominent firm, and for the same reason. It follows that the two supports (of the prominent firm, and the joint support of the competitors) coincide. At the lower bound of that support, the prominent firm sells to everyone, a mass $\phi_1 + \phi_2 + \phi_3$. This firm's profit is $\phi_1 v$, and so that lower-bound price must equal $\bar{p}_3 = \phi_1 v/(\phi_1 + \phi_2 + \phi_3)$.

Consider the interval $[\bar{p}_3, \bar{p}_2)$. No list price is in this interval, and so the there are no atoms. Within this interval there is no gap within the support of the prominent firm: if so then there would be a gap in the support of the competitors' strategies, and so the prominent firm could safely (that is, without losing sales) move a price from the bottom of the gap upward, and so strictly gain. Similarly, there is no gap with the common support of the opponents' strategies. Given that this is so, at least one player $i \in \{2, 3\}$ is willing to set $p_i = \bar{p}_2$ and so earn a payoff of least $\phi_i \bar{p}_3$. Both i and $j \neq i$ face the same same relative payoffs when pricing against the prominent firm, and so $j \neq i$ can also guarantee a payoff of at least $\phi_j \bar{p}_3$ by setting $p_j = \bar{p}_2$.

Player 3 earns $\phi_3 \bar{p}_3$ on the equilibrium path, and at least that payoff by playing $p_3 = \bar{p}_2$ in the deviant subgame. Recall that the prominent player never prices just above $\bar{p}_2$. Hence, player 3 can safely raise price above $\bar{p}_2$ without losing sales, and so earn strictly more than $\phi_3 \bar{p}_3$. We conclude that any equilibrium in this subgame yields a profitable deviation.

The remaining claim of the proposition concerns $n > 3$. Order firms inversely to list price. The "no undercutting" arguments yield the list price stated. Consider the list price of $\bar{p}_i$. The most prominent firm could undercut this and capture the first i types of customers. Hence

$$\phi_1 \bar{p}_1 \geq \left(\sum\nolimits_{j=1}^{i} \phi_j \right) \bar{p}_i \quad \Leftrightarrow \quad \bar{p}_i \leq \frac{\phi_1 \bar{p}_1}{\sum_{j=1}^{i} \phi_j} = \frac{\bar{p}_1}{i}. \tag{41}$$

Each firm can safely raise its list price upward until these constraints bind, and so $\bar{p}_1 = v$ and $\bar{p}_i = v/i$ as stated. No player has an incentive to charge a lower list price in the first stage or undercut in the second stage. It remains to check for an upward deviation of list price.

Suppose that firm i raises its list price to $\tilde{p}_i > \bar{p}_i$. Consider this strategy profile in the subgame.

Firm 1 mixes continuously from $\bar{p}_i$ up to $\tilde{p}_i$ using the distribution function

$$F_1(p) = 1 - \frac{\bar{p}_i}{p}, \tag{42}$$

and places all remaining mass at $\bar{p}_1 = v$.

Any firm $j > i$ which necessarily satisfies $\bar{p}_j < \bar{p}_i$ sets a price equal to list: $p_j = \bar{p}_j$.

Any firm $j < i$ which satisfies $\bar{p}_j \geq \tilde{p}_i$ also sets a final retail price equal to list price: $p_j = \bar{p}_j$.

Any other firm $j \neq i$ necessarily satisfies $\bar{p}_i < \bar{p}_j < \tilde{p}_i$. Such a firm mixes continuously from $\bar{p}_{j+1}$ up to $\bar{p}_j$ using the distribution function

$$F_j(p) = \frac{(j+1)p - v}{p}. \tag{43}$$

Finally, consider the deviant firm i. Take the lowest index k (and so highest list price $\bar{p}_k$) which satisfies $\bar{p}_k < \tilde{p}_i$. Firm i mixes continuously from $\bar{p}_k$ to $\tilde{p}_i$ using the distribution function

$$F_i(p) = \frac{kp - v}{p}, \tag{44}$$

and places all remaining mass at the deviant list price $\tilde{p}_i$.

These strategies generate a Nash equilibrium of the subgame. To see this, note that the most prominent firm always prices at $\bar{p}_i$ or above, and so any $j > i$ has no reason to discount below $\bar{p}_j$. For any other firm j, a price $\bar{p}_i \leq p \leq \tilde{p}_i$, which lies within the support of $F_1(\cdot)$ yields an expected payoff equal to $\phi\bar{p}_i$. If $\bar{p}_j \leq \tilde{p}_i$ then a firm can do no better than this, and so optimally plays the prescribed strategy. If $\bar{p}_j > \tilde{p}_i$ then j is strictly better off by setting $p_j = \bar{p}_j$ and so once again plays optimally. It remains to check the optimality of the most prominent firm. Consider, for example, a price p satisfying $\bar{p}_{j+1} < p < \bar{p}_j$ for some j satisfying $i < j \leq k$. From the perspective of the most prominent firm, this wins the business of the ϕ customers who see only the most prominent firm, the $(j-2)\phi$ competitors index strictly below j, and the customers who see the price of firm i; a total mass of $j\phi$ customers. Additionally, this price p wins those buyers who see firm j's price with probability $1 - F_j(p)$. The expected payoff is

$$p\phi\left[j + (1 - F_j(p))\right] = p\phi\left[j + 1 - \frac{(j+1)p - v}{p}\right] = \phi v, \tag{45}$$

which is the payoff that the firm obtains by serving only captive customers. Finally, the same argument applies when considering any price $\bar{p}_k \leq p \leq \tilde{p}_i$. □

Proof of Proposition 3. In the text we reported the prominence fee that could be charged to the largest firm. Consider instead the fee that could be charged to the intermediate sized firm. This firm earns $\phi_2 v$ in a position of prominence. If it loses prominence, then the worst-case scenario is for prominence to be given (just as before) to the smallest firm, in which case firm 2 earns $\phi_2\phi_3/(\phi_1 + \phi_2 + \phi_3)$. Taking the difference yields a prominence fee that is strictly less than the fee charged to the largest firm. A related calculation (in which the smallest firm is threatened with the prominence of the intermediate sized firm) also generates a smaller prominence fee. □

Proof of Lemma 3. For now, label firms so that $\bar{p}_1 > \cdots > \bar{p}_n$. (The argument in the text confirms that prices are distinct.) If firms maintain list prices on the equilibrium path then firm i earns a profit $\bar{p}_i\alpha_i \prod_{k>i}(1 - \alpha_k)$. Undercutting a cheaper firm $j > i$ earns (arbitrarily close to) $\bar{p}_j\alpha_i \prod_{k>j}(1-\alpha_k)$. Comparing these terms yields the inequality $\bar{p}_j \leq \bar{p}_i \prod_{j\geq k>i}(1-\alpha_k)$. This inequality is satisfied if and only if $\bar{p}_i \leq (1 - \alpha_i)\bar{p}_{i-1}$ for every $i > 1$.

This holds as an equality. If not then we find the lowest i such that the inequality is strict. Such a firm i can locally raise its list price (and final retail price) without violating any "no undercutting" inequalities in the second stage, and so without losing any sales; a contradiction. We conclude that $\bar{p}_i = (1-\alpha_i)\bar{p}_{i-1}$ for $i > 1$. The argument in the text shows that $\bar{p}_1 = v$. Repeated substitution of the relevant equality yields $\bar{p}_i = v\prod_{j=2}^{i}(1-\alpha_j)$ for $i > 1$.

Finally, we confirm the ordering of the firms. Firm $i > 1$ earns profit $v\prod_{j=2}^{i}(1-\alpha_j)$ with probability $\alpha_i\prod_{j>i}(1-\alpha_j)$, yielding an expected profit of $v\alpha_i\prod_{j=2}^{n}(1-\alpha_j)$. Pricing at v would (at worst) yield a profit of $v\alpha_i\prod_{j\neq i}(1-\alpha_j)$. Comparing this terms, necessarily $1-\alpha_i \geq 1-\alpha_1$, or equivalently $\alpha_1 \geq \alpha_i$. We conclude that the largest firm must charge the highest price. □

Proof of Lemma 4. We write $p_i^\star$ for the list prices that satisfy Lemma 3 and generate profits $\pi_i = v\alpha_i\prod_{j=2}^{n}(1-\alpha_j)$. Consider an upward deviation in list price by firm $i > 1$. Suppose that this deviant list price satisfies $p_i^\star < \bar{p}_i \leq p_{i-1}^\star$. Consider the following strategy profile: all firms $j < i-1$ and $j > i$ charge their list prices, so that $p_j = \bar{p}_j = p_j^\star$; firms $j = i-1, i$ mix continuously over the interval $[p_i^\star, \bar{p}_i)$ with distribution satisfying

$$F_j(p) = \frac{1}{\alpha_j}\left[1 - \frac{p_i^\star}{p}\right], \tag{46}$$

and $i-1$ places residual mass at $\bar{p}_{i-1}$. (It is readily verified that $F_j(p)$ satisfies $F_j(p_i^\star) = 0$, is increasing, satisfies $F_j(p) \leq 1$ for $p \leq \bar{p}_j$, and that there are no profitable deviations.) These strategies constitute a Nash equilibrium.

Deviations by i to higher list prices are handled as follows. For example, if i to deviates to a list price satisfying $p_{i-i}^\star < \bar{p}_i \leq p_{i-2}^\star(1-\alpha_i)^{1/2}$ then players $j = i-1, i$ mix by (46) and $i-1$ places residual mass at $\bar{p}_{i-1}$. If however, $p_{i-2}^\star(1-\alpha_i)^{1/2} < \bar{p}_i \leq p_{i-2}^\star$, $j = i-2, i-1, i$ mix in the interval $[p_i^\star, p_{i-1}^\star)$ according to

$$F_j(p) = \frac{1}{\alpha_j}\left[1 - \left(\frac{p_i^\star}{p}\right)^{1/2}\right], \tag{47}$$

while firms $j = i-2, i$ also mix in the interval $[p_{i-2}^\star(1-\alpha_i)^{1/2}, \bar{p}_i)$ according to

$$F_j(p) = \frac{1}{\alpha_j}\left[1 - \frac{p_i^\star}{p(1-\alpha_{i-1})}\right], \tag{48}$$

and place remaining mass at $\bar{p}_j$. (It is readily checked that these strategies yield the payoffs π_j and there are no profitable deviations.) These strategies constitute a Nash equilibrium. One can then consider all higher list-price deviations by firm i iteratively, concluding in all cases that i's profit is π_i, and hence that i has no profitable upward deviation in its list price $\hat{p}_i$. □

Proof of Proposition 4. This follows from Lemmas 3–4 and the arguments in the main text. □

Proof of Proposition 5. We show there is an equilibrium where firm 1 chooses $\alpha_1^\star > \max_{i\neq 1}\{\alpha_i^\star\}$. Equilibria of the pricing subgames are those of Proposition 4. Advertising choices satisfy the first-order conditions (21) and so the remaining deviation checks are non-local:

(i) 1 deviates to $\hat{\alpha}_1 \leq \alpha_j^\star$ where $j\colon \alpha_j^\star = \max_{i>1}\{\alpha_i^\star\}$. Firm j satisfies a first-order condition at $\alpha_j^\star$. Therefore, the best such deviation for 1 is to $\hat{\alpha}_1 = \alpha_j^\star$ (1's revenue (cost) curve is the same (flatter) for $\hat{\alpha}_1 \in [0, \alpha_j^\star]$ than j's over the same interval when $\alpha_i = \alpha_i^\star$ for $i \neq j$). By continuity and 1's first-order condition, 1's profit at any $\hat{\alpha}_1 \geq \alpha_j^\star$ is less than at $\alpha_1^\star$.

(ii) $i > 1$ deviates to $\hat{\alpha}_i \geq \alpha_1^\star$. Firm 1 satisfies their first-order condition at $\alpha_1^\star$. Therefore, the best such deviation for $i > 1$ is to $\hat{\alpha}_i = \alpha_1^\star$ (i's revenue (cost) curve is flatter (steeper) for $\hat{\alpha}_i \in [\alpha_1^\star, 1]$ than 1's over the same interval when $\alpha_i = \alpha_i^\star$ for $i > 1$). But by continuity and i's first-order condition, i's profit at any $\hat{\alpha}_i \leq \alpha_1^\star$ is less than at $\alpha_i^\star$. □

Proof of Proposition 6. We look for equilibria with pure-strategy advertising choices where following any profile of advertising intensities ($\alpha_i^\star$ for $i = 1, \ldots, n$), the equilibrium of the pricing subgame is that of Proposition 4. We put aside trivial equilibria where more than one firm chooses $\alpha_i = 1$ which lead to marginal cost pricing. All firms have zero costs and are therefore symmetric. It follows that although the profile of equilibrium advertising choices we report is unique, the assignment of firms is not. Subject to this disclaimer, the main text explains that one firm will advertise with the outright highest intensity, and we label this firm 1.

By (18) the profit of firm 1 is strictly (and linearly) increasing in α_1 for any $\alpha_i < 1$ for $i > 1$, hence $\alpha_1^\star = 1$. Given $\alpha_1^\star = 1$, (18) shows that the profit of the non-largest firms is maximized at $\alpha_i = 1/2$ for any $\alpha_j < 1$ where $j \neq 1, i$, hence $\alpha_i^\star = 1/2$ for $i > 1$. □

Proof of Proposition 7. We look for equilibria with pure-strategy advertising choices where following any profile of advertising intensities ($\alpha_i^\star$ for $i = 1, \ldots, n$), the equilibrium of the pricing subgame is that of Proposition 4. All firms have the same cost function and are therefore symmetric. It follows that although the profile of equilibrium advertising choices we report is unique, the assignment of firms is not. Subject to this disclaimer, the main text explains that one firm will advertise with the outright highest intensity, and we label this firm 1.

Firms $i, j > 1$ must satisfy their first-order conditions given in (21) but with $C_i = C$. Taking the ratio of i's and j's condition yields

$$\frac{C'(\alpha_i)}{C'(\alpha_j)} = \frac{(1-2\alpha_i)(1-\alpha_j)}{(1-2\alpha_j)(1-\alpha_i)}. \tag{49}$$

If $\alpha_i > (<)\alpha_j$ the LHS $> 1 (< 1)$ but the RHS $< 1 (> 1)$. However, if $\alpha_i = \alpha_j$, (49) is satisfied. Hence $\alpha_i^\star = \alpha_j^\star$. Letting $C(\alpha) = -\log(1-\alpha)$ gives (23), the solution to which gives the values of $\alpha_1^\star$ and $\alpha_i^\star$ for $i > 1$, and that $\alpha_1^\star = 2\alpha_i^\star$. Similar reasoning to that in the proof of Proposition 5 rules out profitable non-local deviations. □

Proof of Proposition 8. Solving eq. (25) yields the expression for μ_1 and for the price $p^\dagger$. Checking that μ_1 lies in the unit interval generates the inequalities reported. □

Proof of Proposition 9. Setting $\lambda_S = 0$ for simplicity (it is straightforward to incorporate $\lambda_S > 0$) the fractions of buyers who are captives for firm i and pairwise shoppers between two firms

i and j are, from eq. (24), $\lambda_i = \mu_1/n$ and $\lambda_{ij} = 2\mu_2/[n(n-1)]$. For well-rehearsed reasons, the "no undercutting" constraint of eq. (27) holds as an equality. Hence

$$\frac{\bar{p}_{i+1}}{\bar{p}_i} = \frac{\lambda_i + \sum_{j<i}\lambda_{ij}}{\lambda_i + \lambda_{i(i+1)} + \sum_{j<i}\lambda_{ij}} = \frac{(\mu_1/n) + 2(i-1)\mu_2/[n(n-1)]}{(\mu_1/n) + 2i\mu_2/[n(n-1)]}$$
$$= \frac{(n-1)\mu_1 + 2(i-1)(1-\mu_1)}{(n-1)\mu_1 + 2i(1-\mu_1)}. \quad (50)$$

Setting $\bar{p}_1 = v$ (for the most expensive firm) and repeated substitution yields the solution for $\bar{p}_i$. By construction, no firm wishes to choose a lower price in either stage. It remains to check for an upward deviation of list price by some firm i. We proceed as before, by constructing a mixed-strategy equilibrium in the deviant subgame which yields a payoff to firm i equal to that received by remaining on the equilibrium path. We omit the details here.

The comparative-static claim regarding the effect of μ_1 (or μ_2) on prices holds by inspection.

For the median price, suppose that n is odd. The median firm i satisfies $i - 1 = (n-1)/2$. Applying the pricing solution for this firm yields $\bar{p}_i = \mu_1 v$ as claimed.

The final claims follow because each firm is indifferent between the specified list price and charging v to captives, and because the average price paid equals industry profit. □

Proof of Proposition 10. For $n = 2$, setting κ equal to the gain from the second quotation:

$$\kappa = \frac{v - \bar{p}_2}{2} = \frac{v(1-\mu_1)}{2-\mu_1} \quad \Leftrightarrow \quad \mu_1 = \frac{v - 2\kappa}{v - \kappa}, \quad (51)$$

which is reported via $1 - \mu_1$ in the proposition. Regarding the average price charged,

$$\bar{p}_1 = v \quad \text{and} \quad \bar{p}_2 = \frac{\mu_1 v}{2 - \mu_1} = v - 2\kappa \quad \Rightarrow \quad \frac{\bar{p}_1 + \bar{p}_1}{2} = v - \kappa. \quad (52)$$

For $n = 3$,

$$\bar{p}_1 = v, \quad \bar{p}_2 = \mu_1 v, \quad \text{and} \quad \bar{p}_3 = \frac{\mu_1 v}{2 - \mu_1}. \quad (53)$$

Now consider the marginal benefit of the second quotation. With probability 1/3 the first quotation yielded the highest price, and so the second quotation is sure to yield a saving of $\bar{p}_1 - \bar{p}_2$. With probability 2/3 the first quotation did not find the cheapest price. The second quotation has probability 1/2 of moving from the intermediate to the lowest price. Hence

$$\kappa = \frac{\bar{p}_1 - \bar{p}_2}{3} + \frac{1}{2}\frac{2(\bar{p}_2 - \bar{p}_3)}{3} = \frac{\bar{p}_1 - \bar{p}_3}{3} = \frac{2}{3}\frac{(1-\mu_1)v}{2-\mu_1}. \quad \Rightarrow \quad \mu_1 = \frac{2v - 6\kappa}{2v - 3\kappa} < \frac{v - 2\kappa}{v - \kappa}, \quad (54)$$

which is equivalent to the statement given in the proposition. Substituting back into prices,

$$\bar{p}_1 = v, \quad \bar{p}_2 = \frac{2v - 6\kappa}{2v - 3\kappa}v, \quad \text{and} \quad \bar{p}_3 = v - 3\kappa \quad \Rightarrow$$
$$\frac{\bar{p}_1 + \bar{p}_2 + \bar{p}_3}{3} = \frac{2v^2 - 6\kappa v + 3\kappa^2}{2v - 3\kappa} = \frac{(2v - 3\kappa)(v - \kappa) - \kappa v}{2v - 3\kappa} = v - \kappa - \frac{\kappa v}{2v - 3\kappa} < v - \kappa. \quad (55)$$

□

Proof of Lemma 7. Consider the prices reported in eq. (28). These satisfy $\bar{p}_0 = v$ and $\bar{p}_n = \mu_1 v/(2-\mu_i)$. Take any price p within the interval bounded by these highest and lowest prices, and write $F_n(p)$ for the cumulative distribution function of prices. For finite n,

$$F_n(p) = \frac{n-i}{n} \quad \Leftrightarrow \quad \bar{p}_i > p \geq \bar{p}_{i+1} \quad \Leftrightarrow$$
$$\frac{\mu_1(n-1)v}{\mu_1(n-1)+2(1-\mu_1)(i-1)} > p \geq \frac{\mu_1(n-1)v}{\mu_1(n-1)+2(1-\mu_1)i}$$
$$\frac{i-1}{n-1} < \frac{\mu_1}{1-\mu_1}\frac{v-p}{2p} \leq \frac{i}{n-1} \quad \Leftrightarrow \quad i = \left\lceil (n-1)\frac{\mu_1}{1-\mu_1}\frac{v-p}{2p} \right\rceil, \tag{56}$$

where "$\lceil \cdot \rceil$" means "the least integer weakly greater than." Hence

$$F_n(p) = 1 - \frac{1}{n}\left\lceil (n-1)\frac{\mu_1}{1-\mu_1}\frac{v-p}{2p} \right\rceil, \tag{57}$$

converges to $F(p)$ (as reported in the lemma) as $n \to \infty$. The distribution of the minimum of two random draws from $F(\cdot)$ is of course $F_{\min}(p) = 1-(1-F(p))^2$ which yields the expression reported. Taking expectations straightforwardly generates the remaining claims. □

Proof of Proposition 11. Equation (34) in the main text is the equilibrium condition. As noted there, the right-hand side is increasing and then decreasing in μ_1. Hence (as stated) κ needs to be smaller enough to be below the maximum of this function, and when it is strictly below the maximum there must be two solutions. □

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